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Introduction:

Background:

The experiment undertaken in this lab involved measuring the frequency response of various first order and second order circuits, which include resistors, capacitors and inductors. This was done by applying and alternating voltage signal across the circuit and measuring the gain at a range of frequencies. By plotting these results on a graph the cut off frequency of the first order circuits where the gain begin to attenuate and the resonant frequency of the second order circuits can be identified.

Aims:

- To accurately measure the frequency response of the first order and second order circuits and compare it to the theoretical calculations.
- To identify the sources of error in the experiment and how it has effected the results.
- Using the frequency response graphs, identify the corner frequency of the first order circuits and the resonant frequency and Q-factor of the second order circuits.

Theory:

Resistors:

Resistors are passive devices which limit the flow of current depending on the voltage across it, and for an ohmic resistor, they follow ohms' law, $V = IR$.

$$Therefore\ \ R = \frac{V}{I}$$

R = Resistance in Ohms, Ω

V = Potential Difference in Volts, V

I = Current in Amperes, A

They have the same characteristics whether the current flowing through them is AC or DC, for a constant resistance, the current flowing through the resistor is directly proportionate to the potential difference (voltage) across it (*figure 1)*. However, for an AC current, the relationship between current and voltage, electrical resistance, is defined as impedance, Z.

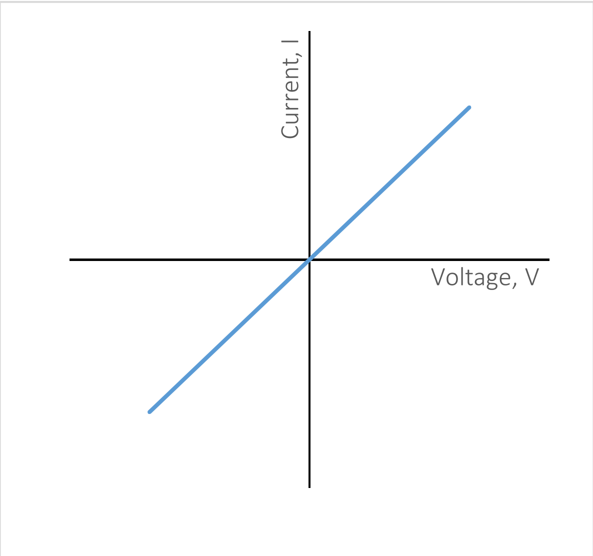

Figure 1: An ohmic resistor IV characteristics for both AC and DC.

- The line passes through the origin of the graph, which signifies that unless there is a potential difference, no current can flow.
- Constant gradient for a fixed resistance.
- $Gradient = \frac{I}{V} = \frac{1}{R}$

Capacitors:

For an alternating current (AC) flowing through the capacitor, the reactance is its resistance depending on the frequency of the AC current.

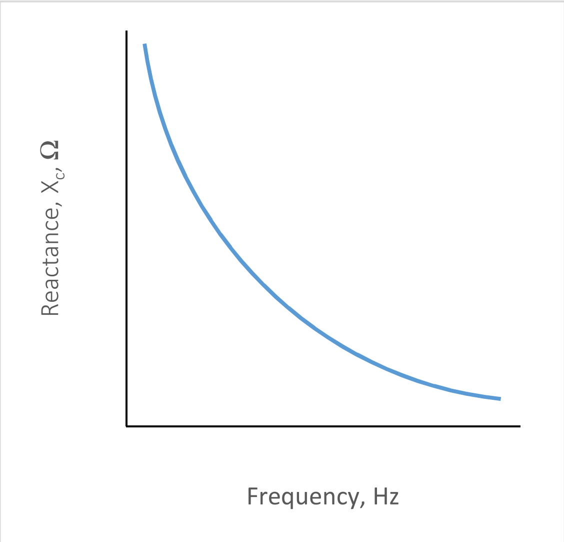

Figure 2: How reactance varies with frequency for a capacitor.

The equation for the reactance of a capacitor is:

$$X_C = \frac{1}{2\pi f C}$$

X_C = Reactance, Ohms, Ω

f = Frequency, Hertz, Hz

C = Capacitance, Farads, F

When the frequency of the AC current is high, the reactance of the capacitor is low, so the resistance is low. Similarly, when the frequency of the AC current is low, the reactance of the capacitor is high, so the resistance is high.

Inductors:

Inductors are coils of wire wrapped around an air or iron core that store energy when a voltage is applied in the form of an electromagnetic field, they also have a small but significant resistance.

For an alternating current flowing through an inductor, the frequency of the current varies the inductors opposition to the current, known as the inductors Reactance.

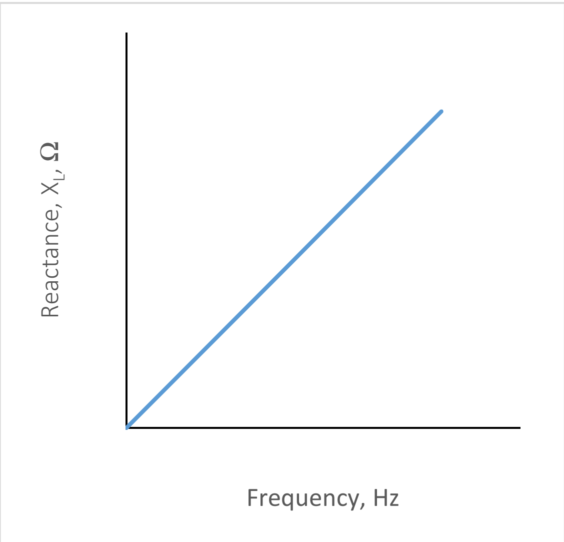

Figure 3: How reactance varies with frequency for an inductor.

The equation for an Inductors Reactance for a given frequency is:

$$X_L = 2\pi f L$$

X_L = Reactance, Ohms, Ω

f = Frequency, Hertz, Hz

L = Inductance, Henries, H

This means that an inductor behaves like a frequency dependant resistor, but opposite to a capacitor.

For a low frequency, the reactance of the inductor is low, so its resistance is low. But for a high frequency, the reactance is high, so its resistance is high.

Therefore, for a constant inductance of an inductor, the frequency of the current flowing though it is directly proportional to the reactance and hence the resistance of the inductor, as shown in *figure 3.*

First Order C-R & L-R Circuits:

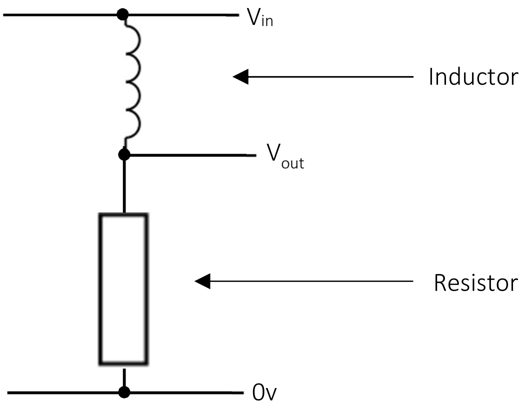

Figure 4a: L-R Low Pass Filter

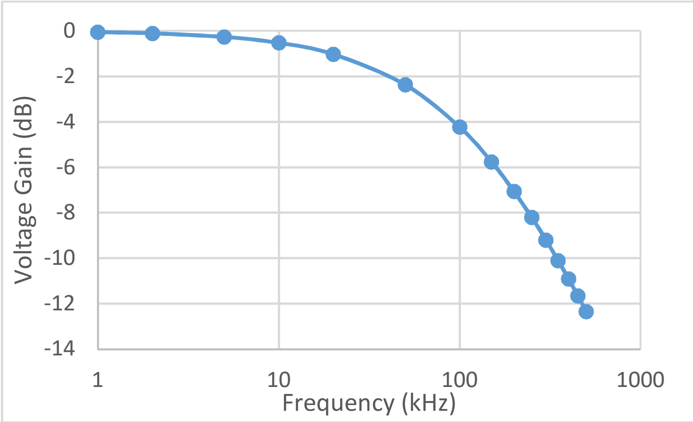

Figure 4b: Theoretical Frequency Response for L-R Series Circuit (L=4.7mH, R=4.7kΩ)

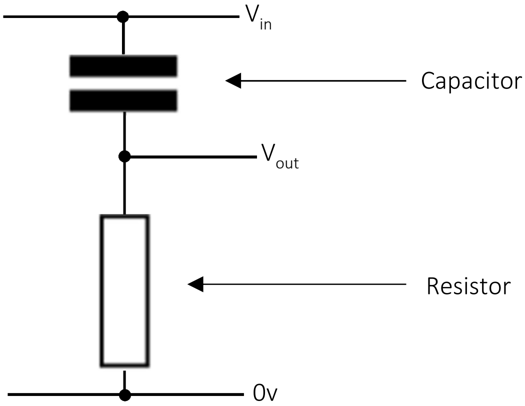

Figure 5a: C-R High Pass Filter

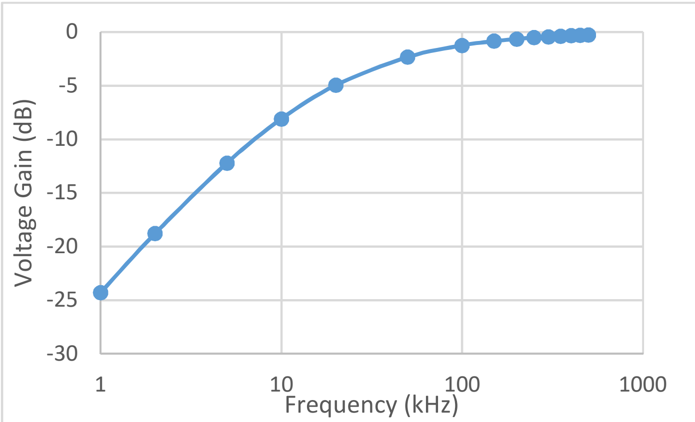

Figure 5b: Theoretical Frequency Response for C-R Series Circuit (C=2.2nF, R=4.7kΩ)

Figure 4a and *5a* represent 2 passive filters, which allow or block signals of certain frequencies to pass.

The cut off frequency of a first order circuit is the frequency at which the reactance of the capacitor or inductor is equal to the resistance of the resistor, $X_C \text{ or } X_L = R$. At this frequency, the gain of the output signal is $20 \log\left(\frac{Vout}{Vin}\right) = -3dB$ or 70.7% of the input signal.

The cut off frequency is useful as it can identify the frequency where the gain begins to attenuate, so that a filter circuit can be designed to rejects certain frequencies above or below a certain point. It is also used in this experiment as part of the calculation to find the resonant frequency of CLR circuits.

For a low pass filter, like in *figure 4b*, frequencies below the cut off frequency are unaffected, whereas frequencies above are attenuated.

For a high pass filter, like in *figure 5b,* frequencies above the cut off frequency are unaffected, whereas frequencies below are attenuated.

Cut off Frequency Equations:

Cut off Frequency for an RL Series Circuit: $f_{CO} = \frac{R}{2\pi L} = \frac{1}{2\pi\tau}$ f_{co} = Cut off Frequency, Hertz, Hz R = Resistance of resistor, Ohms, Ω L = Inductance, Henries, H T = Time constant (R/L), Seconds, s	*Cut off Frequency for an RC Series Circuit:* $f_{CO} = \frac{1}{2\pi RC} = \frac{1}{2\pi\tau}$ f_{co} = Cut off Frequency, Hertz, Hz R = Resistance of resistor, Ohms, Ω C = Capacitance, Farads, F τ = Time constant (RC), Seconds, s

Impedance Equations:

Impedance of an RC Series Circuit: $Z = \sqrt{R^2 + (\frac{1}{\omega C})^2}$ Z = Impedance, Ohms, Ω R = Resistance, Ohms, Ω ω = Angular Frequency (2πf), rad s^{-1} C = Capacitance, Farads, F	*Impedance of an RL Series Circuit:* $Z = \sqrt{R^2 + (\omega L)^2}$ Z = Impedance, Ohms, Ω R = Resistance, Ohms, Ω ω = Angular Frequency (2πf), rad s^{-1} L = Inductance, Henries, H

Time Constant of First Order Circuits:

The time constant of an L-R Series Circuit: $\tau = \frac{L}{R}$ τ = Time Constant, Seconds, s L = Inductance, Henries, H R = Resistance, Ohms, Ω	*The time constant of an C-R Series Circuit:* $\tau = RC$ τ = Time Constant, Seconds, s R = Resistance, Ohms, Ω C = Capacitance, Farads, F

Second Order CLR Circuits:

C-L-R Series Resonant Circuit:

 = Oscilloscope

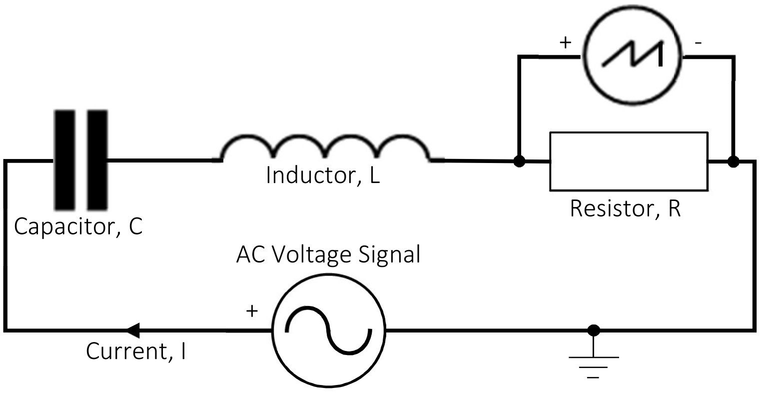

Figure 6a: C-L-R Series Circuit

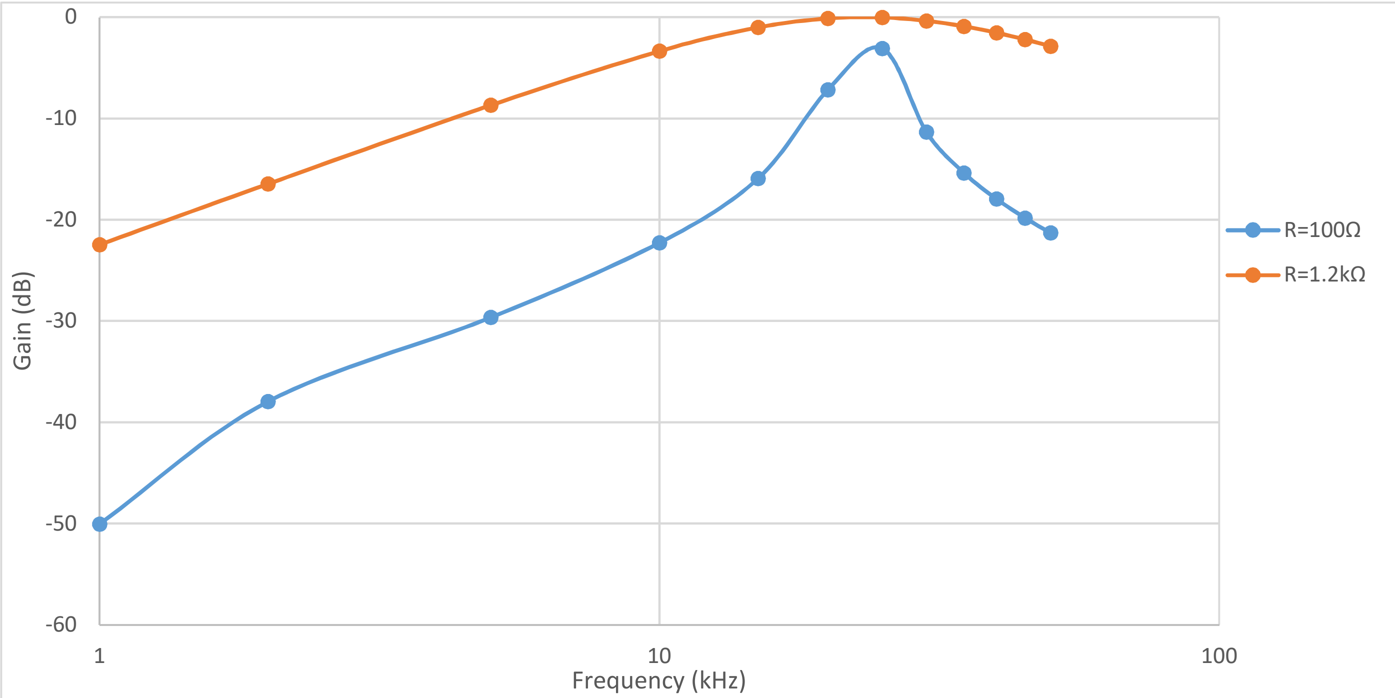

Figure 6b: C-L-R Series Circuit Theoretical Frequency Response
(C=0.01μF, L=4.7mH, R=100Ω, 1.2kΩ)

The circuit in *figure 6a* is a second order circuit because it has 2 components that store energy, the capacitor and the inductor.

Also, seen in *figure 6a,* the current through each component is the same due to being a series circuit and the sum of all the voltages across C, L & R will be equal to the inputs sinusoidal signal voltage. However, when taking into account the phase differences of the voltages across the 3 components, this is not the case. As seen in *figure 7a*, the voltage has no phase shift in relation to the current for the resistor, but the phase shift of the voltage across a capacitor is $\omega = -\pi/2$ and for an inductor is $\omega = \pi/2$ as shown in *figure 7c* and *figure 7b* respectively [1].

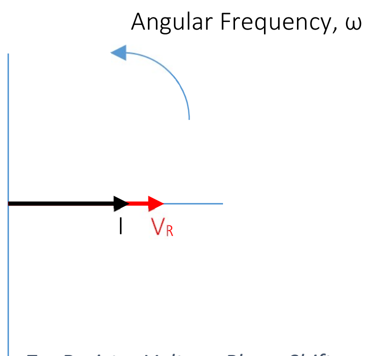

Figure 7a: Resistor Voltage Phase Shift = 0 (in phase)

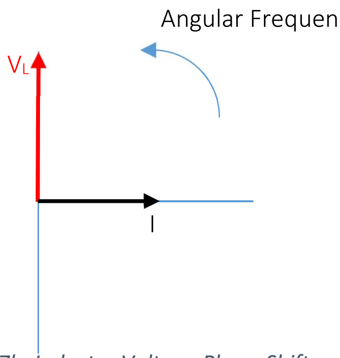

Figure 7b: Inductor Voltage Phase Shift = +90° (V leads the current)

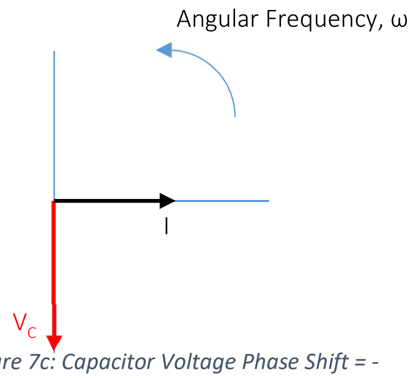

Figure 7c: Capacitor Voltage Phase Shift = -90° (V lags the current)

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In a series C-L-R circuit, the resonant frequency is the frequency where the reactance of the capacitor is equal to the reactance of the inductor, $X_C = X_L$. From the phasor diagrams in *figure 7* the phase difference between the voltage across the inductor and capacitor are 180° out of phase, and since they are the magnitude in voltage, they effectively cancel each other out.

This can also be seen in the impedance equation $V_R = \frac{VR}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$, as when the reactance's of the inductor and capacitor at resonance are equal, $(\omega L - \frac{1}{\omega C})^2$ is effectively zero, so all the voltage is dropped across the resistor.

This can be seen in *figure 6b*, at the peak of the curve, where the gain is at its maximum due to all of the voltage from the supply being dropped across the resistor. From the graph, you can also see that the value of resistor used can affect the shape selectivity of the resonant circuit.

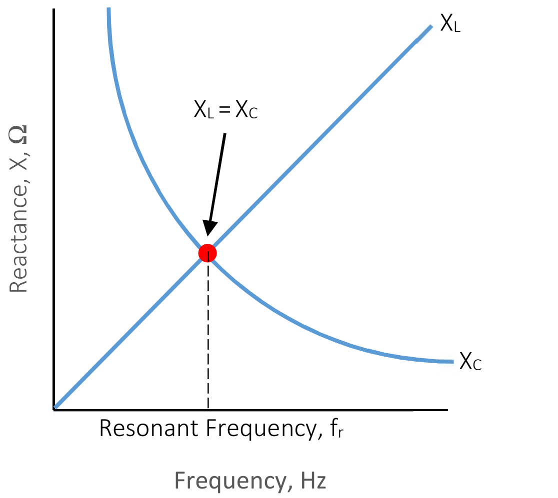

Figure 8: Intersection of reactance at resonance.

Series resonant circuits have very important electronic applications such as noise filtering and selective tuning when used in radio or television to receive different frequencies.

The equation to calculate the resonant frequency of an C-L-R is:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

F_r = Resonant Frequency, Hertz, Hz

L = Inductance, Henries, H

C = Capacitor, Farads, F

Q factor:

The Quality or Magnification factor or an C-L-R circuit is a ratio that compared the current from the supply with the current circulating between L and C in the parallel branch. A higher Q-factor circuit is more selective than a low Q-factor circuit, meaning it can reject frequencies better that are above and below the resonant frequency [2].

The equation to calculate Q-factor is: $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Bandwidth:

The bandwidth is the frequency difference at -3dB of the half power points either side of the resonant frequency, which can be measured on a frequency response graph.

Again, a higher Q-value for bandwidth suggests a greater selectivity.

It can be calculated by: $Q = \frac{f_r}{f_2 - f_1}$

f_r = Resonant Frequency, Hertz, Hz

f_1, f_2 = Half power frequencies, Hertz, Hz

The equation to calculate the impedance of the whole series circuit is:

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

Z = Impedance, Ohms, Ω
R = Resistance, Ohms, Ω
ω = Angular Frequency (2πf), rad s^{-1}
C = Capacitance, Farads, F
L – Inductance, Henries, H

The equation to calculate the voltage across R is:

$$V_R = \frac{VR}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

V_R = Voltage across R, Volts, V
V = Voltage across supply, Volts, V
R = Resistance, Ohms, Ω
ω = Angular Frequency (2πf), rad s^{-1}
C = Capacitance, Farads, F
L – Inductance, Henries, H

These equations were useful in this experiment for calculating the theoretical frequency response of the circuit.

CL-R Parallel Circuit:

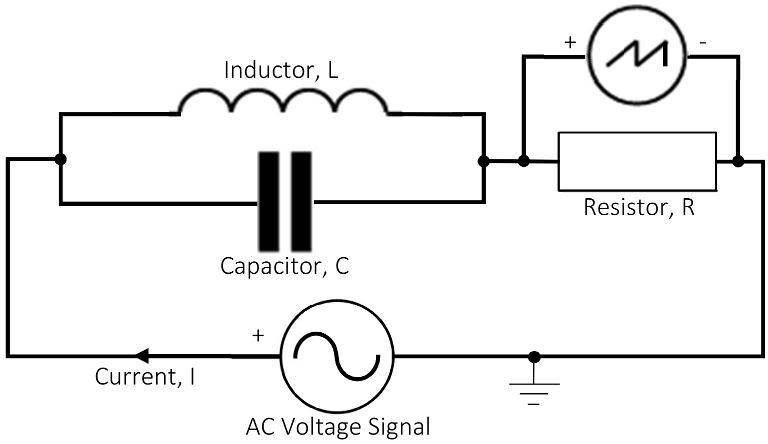

Figure 9a: CL-R Parallel Circuit

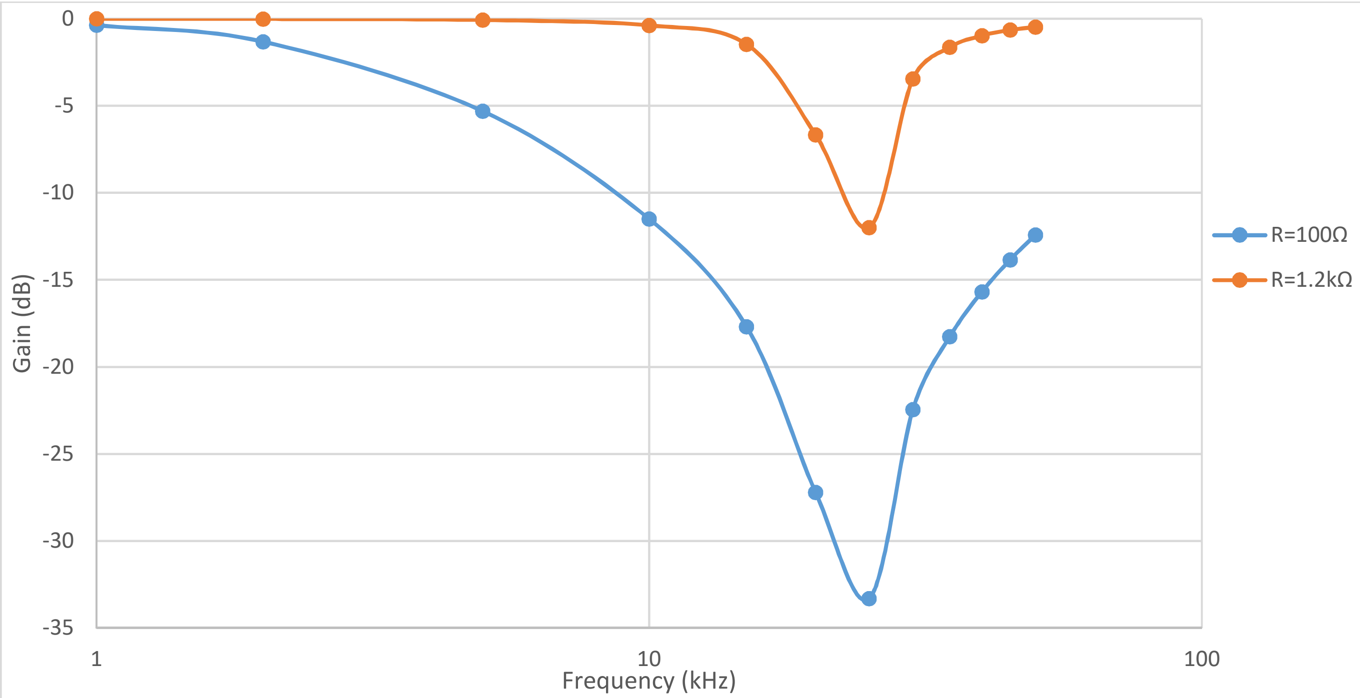

Figure 9b: C-L-R Parallel Circuit (C=0.01µF, L=4.7mH, R=100Ω, 1.2kΩ)

The parallel circuit in *figure 9a* has similar frequency response characteristics as the series circuit seen in *figure 6a*. Due to L and C being in parallel, their combined effective resistance due to reactance is less compared to if they were in series. This can be seen in *figure* as the gain is higher at lower frequencies due to a higher proportion of the voltage drop being across the resistor. At the resonant frequency, no current is drawn from the supply, due to the instantaneous current flowing though L and C are 180° out of phase (equal and opposite), effectively cancelling each other. The result of this is that there will be no voltage drop across the resistor, which can be seen in *figure 9b* as the gain rapidly reduces as it approaches resonance [3].

This can also be seen by using the equation $V_R = \frac{VR}{\sqrt{R^2+(\frac{L}{C})^2\,(\frac{1}{\omega L-\frac{1}{\omega C}})^2}}$, where at resonance the reactance's are equal, so $(\frac{1}{\omega L-\frac{1}{\omega C}})^2 = \frac{1}{0}$, which results in infinite impedance across the parallel junction, so no voltage is dropped across the resistor, hence the large dip in gain at resonance.

The equation to calculate the total impedance is:	The equation to calculate the voltage across R is:
$Z = \sqrt{R^2 + (\frac{L}{C})^2 (\frac{1}{\omega L - \frac{1}{\omega C}})^2}$ Z = Impedance, Ohms, Ω R = Resistance, Ohms, Ω ω = Angular Frequency (2πf), rad s^{-1} C = Capacitance, Farads, F L – Inductance, Henries, H	$V_R = \frac{VR}{\sqrt{R^2 + (\frac{L}{C})^2 (\frac{1}{\omega L - \frac{1}{\omega C}})^2}}$ V_R = Voltage across R, Volts, V V = Voltage across supply, Volts, V R = Resistance, Ohms, Ω ω = Angular Frequency (2πf), rad s^{-1} C = Capacitance, Farads, F L – Inductance, Henries, H

These equations were used in predicting the theoretical frequency response of this circuit.

Method:

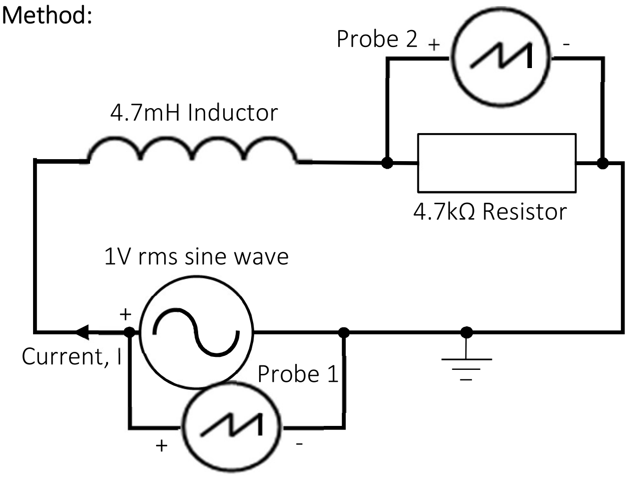

Figure 10a: Series L-R Circuit

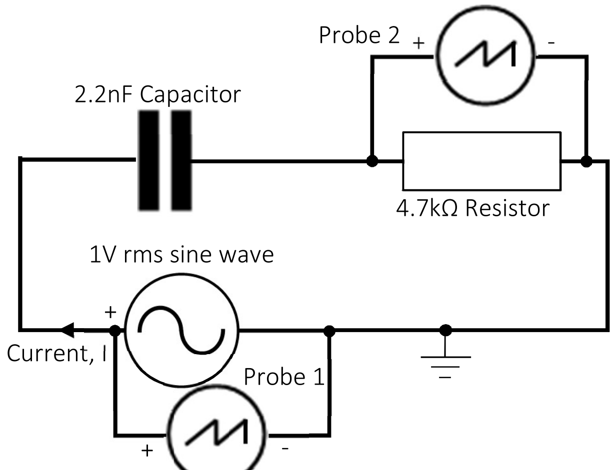

Figure 10b: Series C-R Circuit

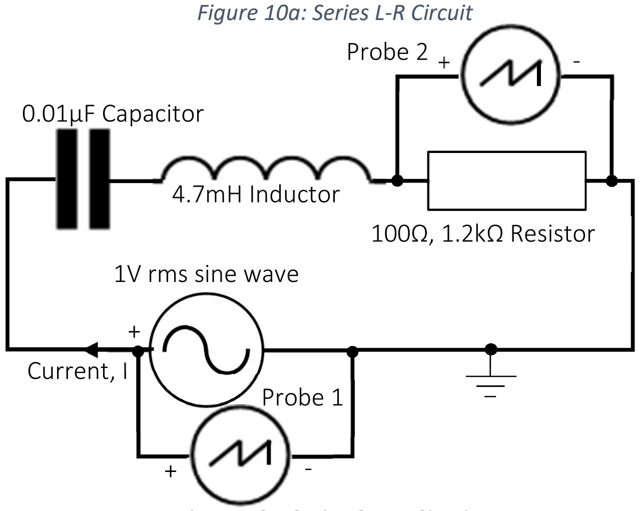

Figure 10c: Series C- L-R Circuit

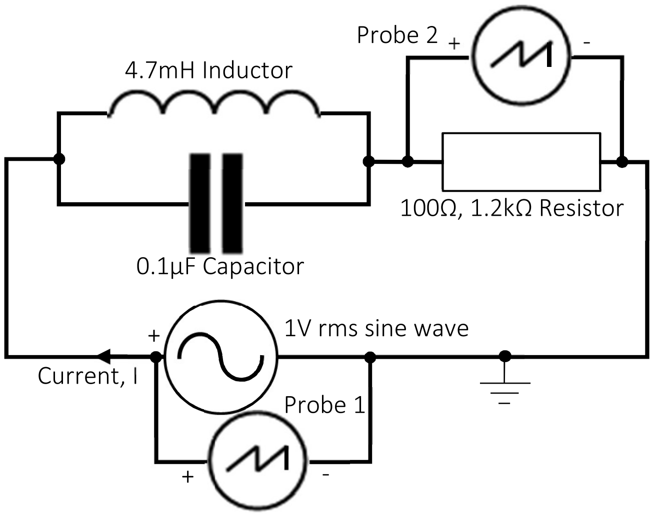

Figure 10d: Parallel LC in series with R circuit

For First Order Circuits:

1) Construct the circuit as shown in *figure 10a*, connect probe 1 of the oscilloscope across the supply and probe 2 across the resistor. (Solder onto matrix board to avoid parasitic capacitance associated with breadboard.)
2) Construct a table shown in *figure 11.*

Frequency (kHz)	V_{in} (Mv)	V_{out} (Mv)	Voltage Gain (V_{out}/V_{in})	Gain (Db)
1				
2				
5				

Figure 11: Example measurements table

3) Set the supply voltage to a 1.000V rms sine wave on the function generator at a frequency of 1kHz.
4) On the oscilloscope, set up a measurement to measure the rms voltage of both probes.
5) Measure the voltage of the input and the output to a precision of 0.01V or 10mV and record this in the table.
6) Repeat step 5 though a range of frequencies up to 500kHz (1, 2, 5, 10, 20, 50 etc), while keeping the voltage constant at 1V rms.
7) Using the equation $gain = {}^{V_{out}}/_{V_{in}}$, calculate the voltage gain using the data measured for each frequency and record this in the table.
8) Using the equation $gain = 20\times\log({}^{V_{out}}/_{V_{in}})$, calculate the gain in Db (decibels) for each frequency. (This is gain that will be used when plotting the data on a graph)
9) Plot a log-linear graph of frequency against gain (dB) on the same graph as the theoretical data for comparison,
10) Repeat steps 1-9 for the remaining first order circuit in *figure 10b*.
11) Identify the corner frequency by drawing a horizontal line at -3dB until it intersects the line, then draw vertical line until it intersects the X axis. Where it intersects the X axis represents the corner frequency.

For Second Order Circuits:

1) Construct the circuit as shown in *figure 10c*, connect probe 1 of the oscilloscope across the supply and probe 2 across the resistor.
2) Construct a table shown in *figure 12.*

Frequency (kHz)	V_{in} (Mv)	V_{out} (Mv)	Voltage Gain (V_{out}/V_{in})	Gain (Db)
1				
2				
5				

Figure 12: Example measurements table

3) Set the supply voltage to a 1.000V rms sine wave on the function generator at a frequency of 1kHz.
4) On the oscilloscope, set up a measurement to measure the rms voltage of both probes.
5) Measure the voltage of the input and the output to a precision of 0.01V or 10mV and record this in the table.
6) Repeat step 5 though a range of frequencies up to 50kHz (1, 2, 5, 10, 12, 15 etc), while keeping the voltage constant at 1V rms.
7) Using the equation $gain = {}^{V_{out}}/_{V_{in}}$, calculate the voltage gain using the data measured for each frequency and record this in the table.
8) Using the equation $gain = 20\times\log({}^{V_{out}}/_{V_{in}})$, calculate the gain in Db (decibels) for each frequency. (This is gain that will be used when plotting the data on a graph)
9) Plot a log-linear graph of frequency against gain (dB) on the same graph as the theoretical data for comparison,
10) Replace the 100Ω resistor with a 1.2kΩ resistor.
11) Repeat steps 1-10 for the remaining second order circuit in *figure 10d*.
12) Identify the resonant frequency by the maximum or minimum point on the curve for each circuit.

Calculations:

L-R Series Circuit Theoretical Frequency:

$$f_{co} = \frac{R}{2\pi L}$$

$$f_{co} = \frac{4700}{2\pi \times 0.0047} = 159.2kHz$$

C-R Series Circuit Theoretical Frequency:

$$f_{co} = \frac{1}{2\pi RC}$$

$$f_{co} = \frac{1}{2\pi \times 4700 \times (2.2\times10^{-9})} = 15.4kHz$$

C-L-R Series/Parallel Circuit Resonant Frequency:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{(0.047) \times (0.01\times10^{-6})}} = 23.2kHz$$

Theoretical Q-factor for C-L-R Circuit:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

R = 100Ω

$$Q = \frac{1}{100}\sqrt{\frac{0.0047}{0.01\times10^{-6}}} = 6.86$$

R=1.2kΩ

$$Q = \frac{1}{1200}\sqrt{\frac{0.0047}{0.01\times10^{-6}}} = 0.57$$

Measured Q-factor for C-L-R Circuit:

Measured off *figure 15e:*

$$Q = \frac{f_r}{f_2 - f_1}$$

R = 100Ω

$$Q = \frac{23000}{27000 - 21000} = 3.83$$

R=1.2kΩ

$$Q = \frac{23000}{54000 - 10500} = 0.53$$

Results:

L-R Series Circuit:

L-R Series Circuit Measured Frequency Response (L=4.7mH, R=4.7kΩ)				
Frequency (kHz)	Vin (mV)	Vout (mV)	Voltage Gain	Gain (dB)
1	2050	2030	0.99	-0.09
2	2050	2030	0.99	-0.09
5	2050	2030	0.99	-0.09
10	2050	2010	0.98	-0.17
20	2050	2010	0.98	-0.17
50	2050	1950	0.95	-0.43
100	2070	1770	0.86	-1.36
150	2090	1530	0.73	-2.71
200	2090	1310	0.63	-4.06
250	2090	1110	0.53	-5.50
300	2090	920	0.44	-7.13
350	2090	760	0.36	-8.79
400	2090	640	0.31	-10.28
450	2090	520	0.25	-12.08
500	2090	420	0.20	-13.94

Figure 13a: L-R Series Circuit Measures Data

L-R Series Circuit Theoretical Frequency Response (R=4.7kΩ, L=4.7mH)				
Frequency (kHz)	Vin (V)	Vout (V)	Voltage Gain	Voltage (dB)
1	1	1.00	1.00	0.00
2	1	1.00	1.00	0.00
5	1	1.00	1.00	0.00
10	1	1.00	1.00	-0.02
20	1	0.99	0.99	-0.07
50	1	0.95	0.95	-0.41
100	1	0.85	0.85	-1.45
150	1	0.73	0.73	-2.76
200	1	0.62	0.62	-4.11
250	1	0.54	0.54	-5.40
300	1	0.47	0.47	-6.58
350	1	0.41	0.41	-7.66
400	1	0.37	0.37	-8.64
450	1	0.33	0.33	-9.54
500	1	0.30	0.30	-10.36

Figure 13b: L-R Series Circuit Theoretical Data

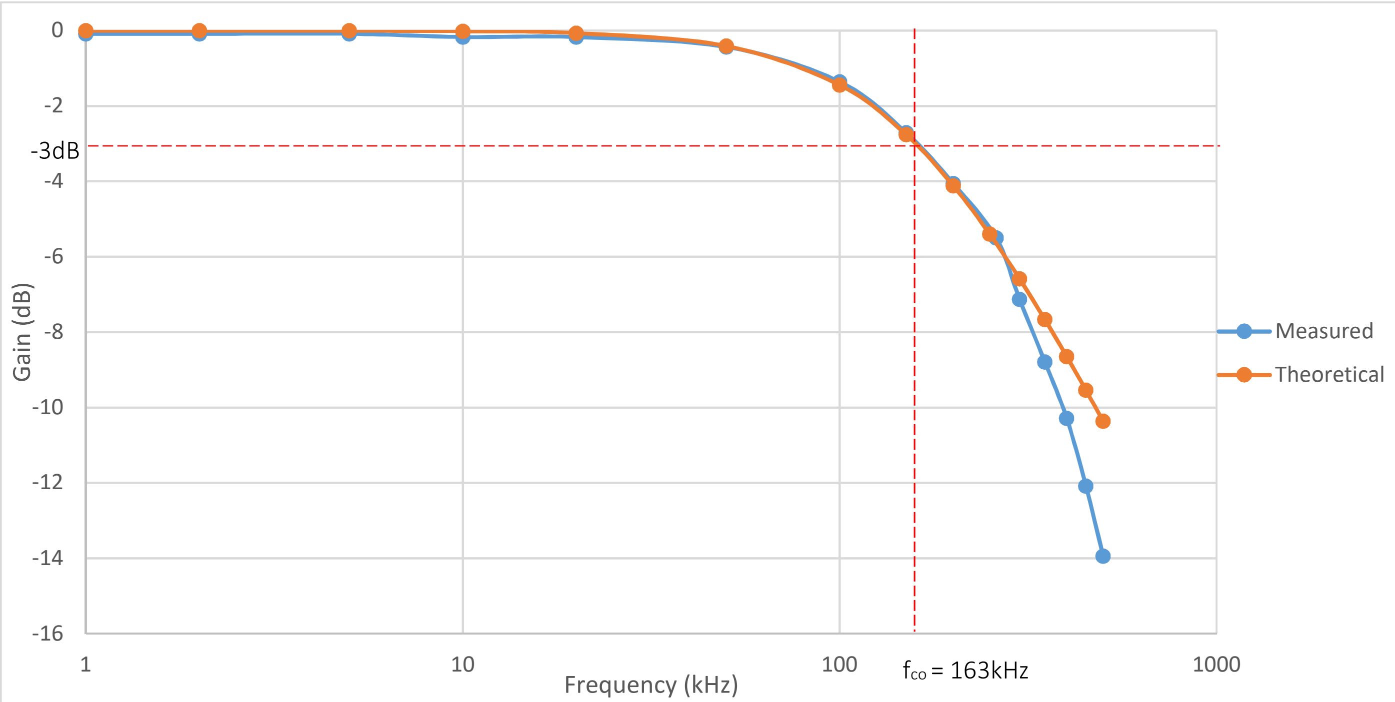

Figure 13c: L-R Series Circuit Frequency Response plot

From *figure 13c*, the shape of the measured frequency response is similar to the theoretical frequency response because as frequency increases beyond the corner frequency, the signal is attenuated. However, the measured results show that the gain begin to attenuate at a faster rate than predicted beyond 300kHz. The circuit also has a slightly higher corner frequency than the theoretical predictions, 163kHz compared to 159.2kHz. This could be due to the inductor having a slight DC resistance as well as resistances from the leads of the components and cables due to the resistivity of the materials they are constructed of. Also, there is parasitic capacitance within the coils of the inductor as well as the board used to construct the circuit, which can be seen in *figure 13d*. Furthermore, manufacturing tolerances in the resistor and inductor will vary the gain of the circuit compared to their ideal values.

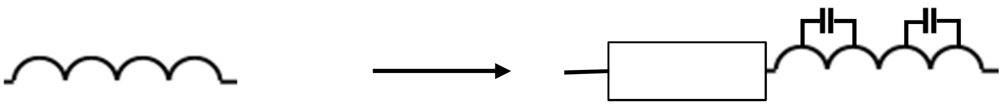

Figure 13d: Ideal Inductor on left, real world inductor on the right.

C-R Series Circuit:

C-R Circuit Measured Frequency Response (C=2.2nF, R=4.7kΩ)				
Frequency (kHz)	Vin (mV)	Vout (mV)	Voltage Gain	Gain (dB)
1	2070	160	0.08	-22.24
2	2050	300	0.15	-16.69
5	2050	640	0.31	-10.11
10	2050	1110	0.54	-5.33
20	2050	1590	0.78	-2.21
50	2050	1930	0.94	-0.52
100	2070	2010	0.97	-0.26
150	2090	2030	0.97	-0.25
200	2070	2030	0.98	-0.17
250	2070	2030	0.98	-0.17
300	2070	2030	0.98	-0.17
350	2070	2030	0.98	-0.17
400	2070	2050	0.99	-0.08
450	2070	2050	0.99	-0.08
500	2070	2050	0.99	-0.08

Figure 14a: C-R Series Circuit Measures Data

C-R Series Circuit Theoretical Frequency Response (R=4.7kΩ, C=2.2nF)				
Frequency (kHz)	Vin (V)	Vout (V)	Voltage Gain	Voltage (dB)
1	1	0.06	0.06	-24.59
2	1	0.12	0.12	-18.61
5	1	0.28	0.28	-10.96
10	1	0.51	0.51	-5.87
20	1	0.76	0.76	-2.35
50	1	0.95	0.95	-0.47
100	1	0.99	0.99	-0.12
150	1	0.99	0.99	-0.05
200	1	1.00	1.00	-0.03
250	1	1.00	1.00	-0.02
300	1	1.00	1.00	-0.01
350	1	1.00	1.00	-0.01
400	1	1.00	1.00	-0.01
450	1	1.00	1.00	-0.01
500	1	1.00	1.00	0.00

Figure 14b: C-R Series Circuit Theoretical Data

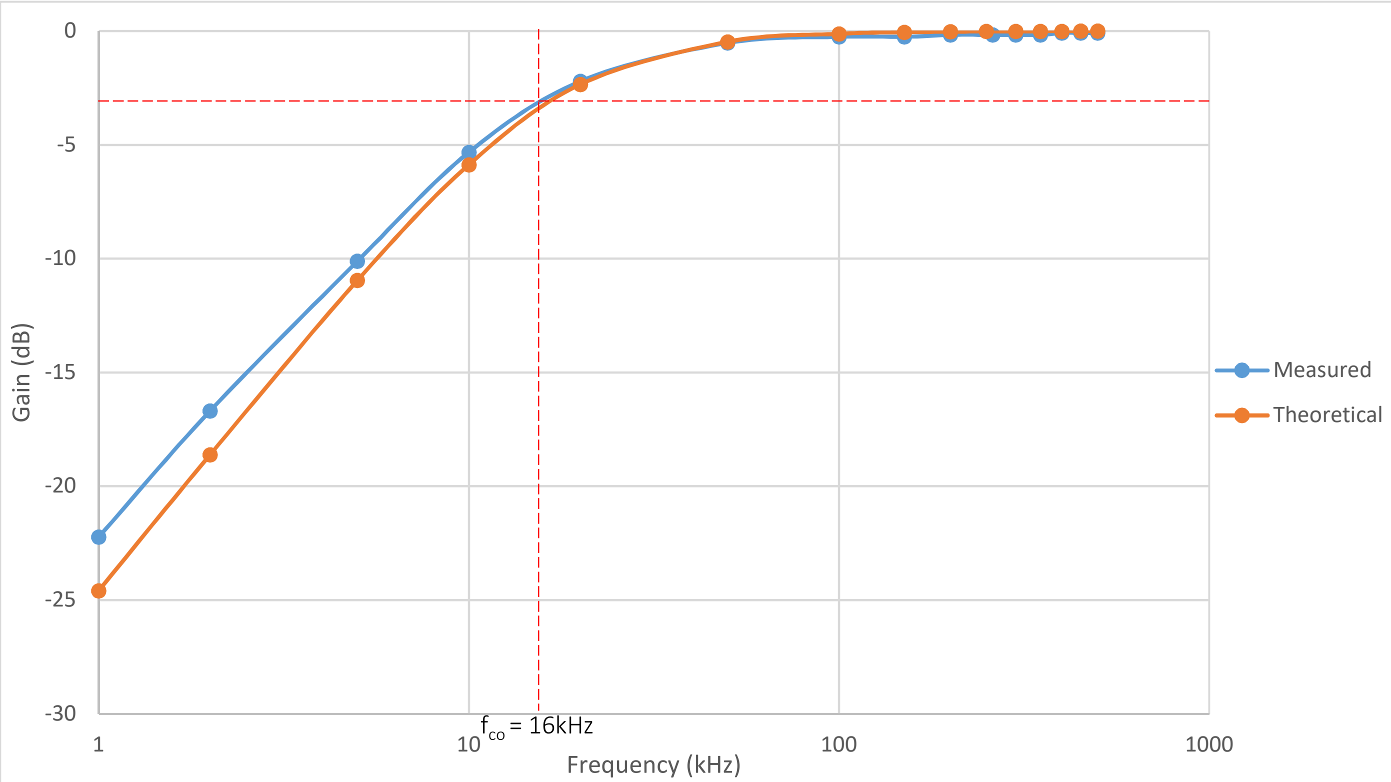

Figure 14c: C-R Series Circuit Frequency Response plot

From *figure 14c*, the shape of the measured frequency response is similar to the theoretical frequency response because as frequency decreases below the corner frequency, the signal is attenuated. However, the measured results show that the gain is slightly higher as the gain increases from the region of 1kHz to 10kHz to what was predicted. The circuit also has a slightly higher corner frequency compared to the theoretical predictions, 16kHz compared to 15.4kHz. The capacitor is not ideal so it will have parasitic inductance as well as parasitic resistance from the resistivity of the materials, which will cause variation in results. The unwanted inductance in the capacitor will slightly reduce the impedance, causing a higher proportion of the supply voltage being dropped across the resistor, hence the gain is slightly higher compared to its theoretical value. A visualisation of what the capacitor is actually like in the circuit can be seen in *figure 14d*. Furthermore, the manufacturing tolerances of the components will cause their real-world values to be slightly different, causing the measured data to not the same as the theoretical.

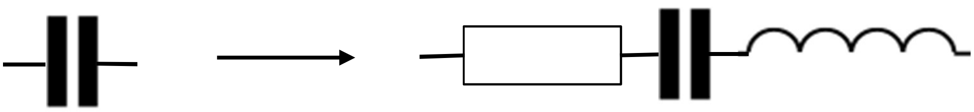

Figure 14d: Ideal capacitor on the left, real world capacitor on the

C-L-R Series Second Order Circuit:

C-L-R Series Circuit Measured (C=0.01µF, L=4.7mH R=100Ω)				
Frequency (kHz)	Vin (mV)	Vout (mV)	Voltage Gain	Gain (dB)
1	1000	11.2	0.01	-39.02
2	1000	15.4	0.02	-36.25
5	1000	32.9	0.03	-29.66
10	1000	73.2	0.07	-22.71
12	1000	96.6	0.10	-20.30
15	1000	148	0.15	-16.59
20	1000	352.5	0.35	-9.06
22	1000	505.2	0.51	-5.93
25	1000	483.4	0.48	-6.31
30	1000	251.5	0.25	-11.99
32	1000	207.5	0.21	-13.66
40	1000	123.4	0.12	-18.17
42	1000	112.2	0.11	-19.00
45	1000	99.3	0.10	-20.06
50	1000	84	0.08	-21.51

Figure 15a: C-L-R Series Circuit Measured Data (R=100Ω)

C-L-R Series Circuit Theoretical (C=0.01µF, L=4.7mH, R=100Ω)				
Frequency (kHz)	Vin (V)	Vr (V)	Voltage Gain	Gain (dB)
1	1	0.01	0.01	-44.02
2	1	0.01	0.01	-37.95
5	1	0.03	0.03	-29.65
10	1	0.08	0.08	-22.28
12	1	0.10	0.10	-19.80
15	1	0.16	0.16	-15.93
20	1	0.44	0.44	-7.17
22	1	0.80	0.80	-1.89
25	1	0.70	0.70	-3.08
30	1	0.27	0.27	-11.35
32	1	0.22	0.22	-13.23
40	1	0.13	0.13	-17.95
42	1	0.12	0.12	-18.76
45	1	0.10	0.10	-19.83
50	1	0.09	0.09	-21.31

Figure 15b: C-L-R Series Circuit Theoretical Data (R=100Ω)

C-L-R Series Circuit Measured (C=0.01µF, L=4.7mH, R=1.2kΩ)				
Frequency (kHz)	Vin (mV)	Vout (mV)	Voltage Gain	Gain (dB)
1	1000	74	0.07	-22.62
2	1000	147	0.15	-16.65
5	1000	360	0.36	-8.87
10	1000	649	0.65	-3.76
12	1000	743	0.74	-2.58
15	1000	844	0.84	-1.47
20	1000	931	0.93	-0.62
22	1000	945	0.95	-0.49
25	1000	948	0.95	-0.46
30	1000	920	0.92	-0.72
32	1000	903	0.90	-0.89
40	1000	819	0.82	-1.73
42	1000	794	0.79	-2.00
45	1000	794	0.79	-2.00
50	1000	713	0.71	-2.94

Figure 15c: C-L-R Series Circuit Measured Data (R=1.2kΩ)

C-L-R Series Circuit Theoretical (C=0.01µF, L=4.7mH, R=1.2kΩ)				
Frequency (kHz)	Vin (V)	Vr (V)	Voltage Gain	Gain (dB)
1	1	0.08	0.08	-22.46
2	1	0.15	0.15	-16.47
5	1	0.37	0.37	-8.69
10	1	0.68	0.68	-3.36
12	1	0.78	0.78	-2.19
15	1	0.89	0.89	-1.02
20	1	0.99	0.99	-0.13
22	1	1.00	1.00	-0.02
25	1	1.00	1.00	-0.03
30	1	0.96	0.96	-0.37
32	1	0.94	0.94	-0.57
40	1	0.84	0.84	-1.54
42	1	0.81	0.81	-1.80
45	1	0.78	0.78	-2.20
50	1	0.72	0.72	-2.86

Figure 15d: C-L-R Series Circuit Theoretical Data (R=1.2kΩ)

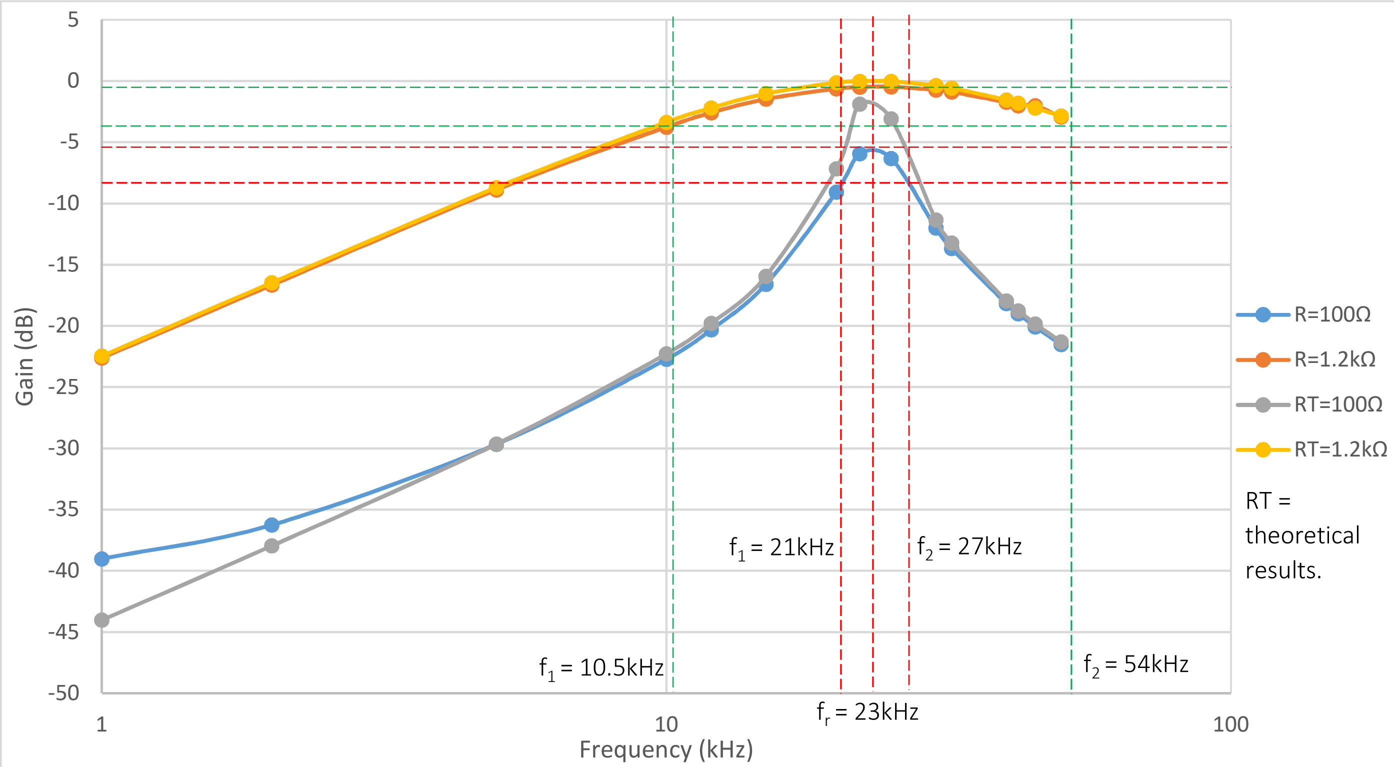

Figure 15e: C-L-R Series Circuit Frequency Response plot.

In *figure 15e* there are frequency response plots of 2 different C-L-R series circuits, one where R=100Ω and the other where R=1.2kΩ. For the R=1.2kΩ circuit, the measured results are almost identical to the theoretical results, with any slight differences, especially at the resonant frequency, probably being due to component tolerances. The resonant frequency for both circuits is almost identical to the theoretical resonant frequency 23kHz compared to 23.2kHz. However, for the R=100Ω circuit, you can see that the gain at resonance is lower than predicted. This could be down to a combination of parasitic resistances and capacitance in the inductor and parasitic inductance in the capacitor, which would cause the voltage across the parallel junction to be slightly higher, making the voltage across R slightly lower. Internal resistances of the electronic measuring equipment and the cables used could also be a factor in why the results are not quite as predicted.

More likely, this could be due to not enough frequency points being plotted during the experiment, as the trend line is not an accurate representation of the frequency responses shape as there are not enough points around the resonant frequency to accurately determine its shape. This error is particularly evident in measuring the Q-value, which is 3.83 compared to the predicted 6.86 for R=100Ω and 0.53 compared to 0.57 for R=1.2kΩ.

The main difference in *figure 15e* is the difference in shape of the 2 circuits with different resistors. You can see that the R=100Ω circuit has a much sharper gain attenuation above and below the resonant frequency, making is more selective than the R=1.2kΩ circuit.

Due to the sharper and more selective gain profile, the R=100Ω circuit has a much higher Q value, making is more useful in applications where only select frequencies need to be allowed through. However, the R=1.2kΩ has a much higher bandwidth due to the broader peak which covers a larger range of frequencies above -3dB.

C-L-R Parallel Second Order Circuit:

CL-R Parallel Circuit Measured (C=0.01µF, L=4.7mH, R=100Ω)				
Frequency (kHz)	Vin (mV)	Vout (mV)	Voltage Gain	Gain (dB)
1	1000	585.5	0.59	-4.65
2	1000	556.7	0.56	-5.09
5	1000	424.5	0.42	-7.44
10	1000	241	0.24	-12.36
12	1000	188.5	0.19	-14.49
15	1000	126.5	0.13	-17.96
20	1000	49	0.05	-26.20
22	1000	26.8	0.03	-31.44
25	1000	23.7	0.02	-32.51
30	1000	68.8	0.07	-23.25
32	1000	86.5	0.09	-21.26
35	1000	112.6	0.11	-18.97
40	1000	153.1	0.15	-16.30
42	1000	168.5	0.17	-15.47
45	1000	189.8	0.19	-14.43
50	1000	224.2	0.22	-12.99

Figure 16a: CL-R Parallel Circuit Measured Data (R=100Ω)

CL-R Parallel Circuit Theoretical (C=0.01µF, L=4.7mH, R=100Ω)				
Frequency (kHz)	Vin (V)	Vr (V)	Voltage Gain	Gain (dB)
1	1	0.96	0.96	-0.36
2	1	0.86	0.86	-1.32
5	1	0.54	0.54	-5.31
10	1	0.27	0.27	-11.51
12	1	0.20	0.20	-13.87
15	1	0.13	0.13	-17.70
20	1	0.04	0.04	-27.21
22	1	0.02	0.02	-36.09
25	1	0.02	0.02	-33.30
30	1	0.08	0.08	-22.45
32	1	0.09	0.09	-20.46
35	1	0.12	0.12	-18.26
40	1	0.16	0.16	-15.68
42	1	0.18	0.18	-14.88
45	1	0.20	0.20	-13.84
50	1	0.24	0.24	-12.42

Figure 16b: CL-R Parallel Circuit Theoretical Data (R=100Ω)

CL-R Series Circuit Measured (C=0.01µF, L=4.7mH, R=1.2kΩ)				
Frequency (kHz)	Vin (mV)	Vout (mV)	Voltage Gain	Gain (dB)
1	1000	74	0.07	-22.62
2	1000	147	0.15	-16.65
5	1000	360	0.36	-8.87
10	1000	649	0.65	-3.76
12	1000	743	0.74	-2.58
15	1000	844	0.84	-1.47
20	1000	931	0.93	-0.62
22	1000	945	0.95	-0.49
25	1000	948	0.95	-0.46
30	1000	920	0.92	-0.72
32	1000	903	0.90	-0.89
40	1000	819	0.82	-1.73
42	1000	794	0.79	-2.00
45	1000	794	0.79	-2.00
50	1000	713	0.71	-2.94

Figure 16c: CL-R Parallel Circuit Measured Data (R=1.2kΩ)

CL-R Series Circuit Theoretical (C=0.01µF, L=4.7mH, R=1.2kΩ)				
Frequency (kHz)	Vin (V)	Vr (V)	Voltage Gain	Gain (dB)
1	1	0.08	0.08	-22.46
2	1	0.15	0.15	-16.47
5	1	0.37	0.37	-8.69
10	1	0.68	0.68	-3.36
12	1	0.78	0.78	-2.19
15	1	0.89	0.89	-1.02
20	1	0.99	0.99	-0.13
22	1	1.00	1.00	-0.02
25	1	1.00	1.00	-0.03
30	1	0.96	0.96	-0.37
32	1	0.94	0.94	-0.57
40	1	0.84	0.84	-1.54
42	1	0.81	0.81	-1.80
45	1	0.78	0.78	-2.20
50	1	0.72	0.72	-2.86

Figure 16d: CL-R Parallel Circuit Theoretical Data (R=1.2kΩ)

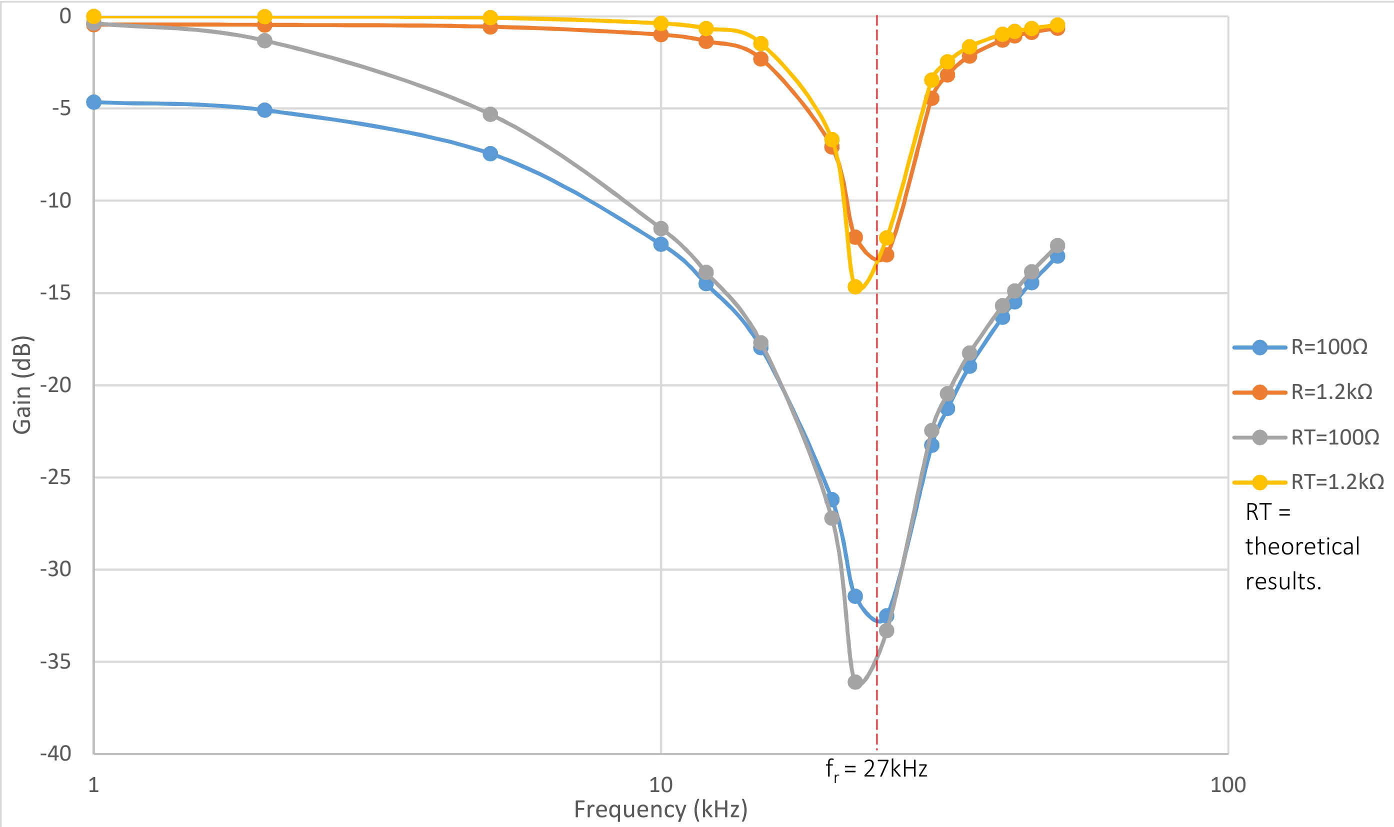

Figure 16e: CL-R Parallel Circuit Frequency Response plot

The differences in the shape of *figure 16e* between the measured results and the theoretical results are very similar to the differences in *figure 15e*. However, a major difference to identify between the measured and theoretical results are the resonant frequencies, with the resonant frequency being 27kHz compared to the predicted 23kHz.

This, along with the slight variations could possibly be due to component tolerances and parasitic capacitance or inductance within the circuit. The effect of this parasitic inductance and capacitance can be seen at the start of the frequency response curve at 1kHz, where the gain predicted is almost 5dB higher than what was measured. Unwanted resistances from the resistivity of the materials used could have been a factor in increasing the total impedance of the parallel junction, resulting in less voltage being lost across the resistor, hence providing a lower gain.

However, the main source of error to why the measured gain at the resonant frequency is not as predicted is due to lack of measured data plots, which would give a more accurate trendline.

Errors:
Overall, the main source of error for the first order circuits is the parasitic inductance, capacitance and material resistances, with possible systematic errors from component tolerances and measuring equipment precision also having an effect.
For the second order circuits, the main source of error is not taking enough gain calculations at suitable frequencies around the resonant frequency, hence an accurate trend line cannot be drawn. The other sources of error include the effects of parasitic inductance, capacitance and resistance as well as tolerances/precision in the components and measuring equipment.

Conclusion:

The frequency response of both first order and second order circuits have been successfully measured and as seen from the graphs, are very close to their theoretical predictions. From these graphs, the corner frequency was measured for the first order circuits, and the resonant frequency was measured for the second order circuits, with the main source of error being not taking a high enough concentration of measurements around the resonant frequency.

From the frequency response graphs, it was identified that an L-R series circuit produces a low-pass filter while a C-R series circuit produces a high-pass filter. It was also identified that for the second order systems, R=100Ω produced a higher Q-factor and hence is more selective compared to R=1.2kΩ.

References:

[1] Electronics Tutorials. (2016). Series Resonance Circuit, [Online]. Available: http://www.electronics-tutorials.ws/accircuits/series-resonance.html

[2] J O Bird, *Electrical and electronic principles and technology*, Fifth edition. London; New York: Routledge, Taylor & Francis Group, 2014.

[3] Electronics Tutorials. (2016). Parallel Resonance Circuit [Online]. Available: http://www.electronics-tutorials.ws/accircuits/parallel-resonance.html