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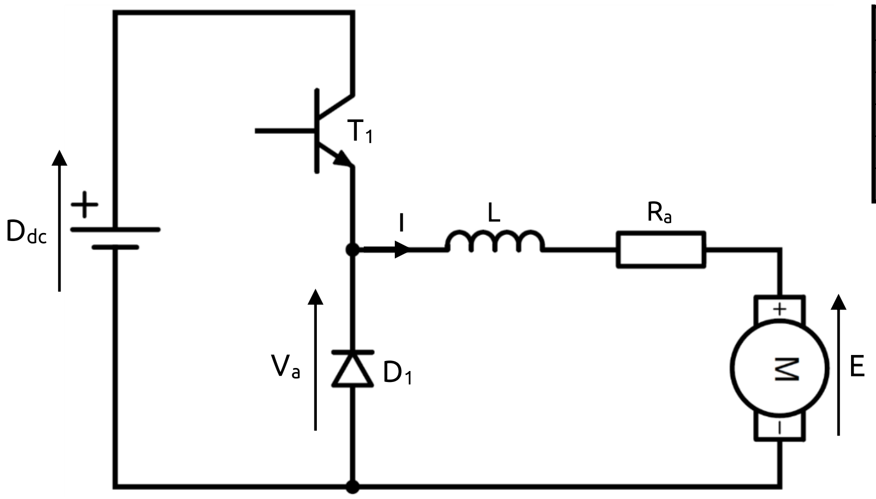
EEE223 ASSIGNMENT 2

The motor controller circuit is shown in *figure 1*.



V_{dc}	60V
L	4mH
ψ	0.08Nm/A
T_1 on-voltage	0.8V
D_1 on-voltage	1V
Winding resistance	0.1Ω

Figure 1: Speed controller circuit.

Part A

Shown below in *figure 2* is the circuit representation for T1 being switched on.

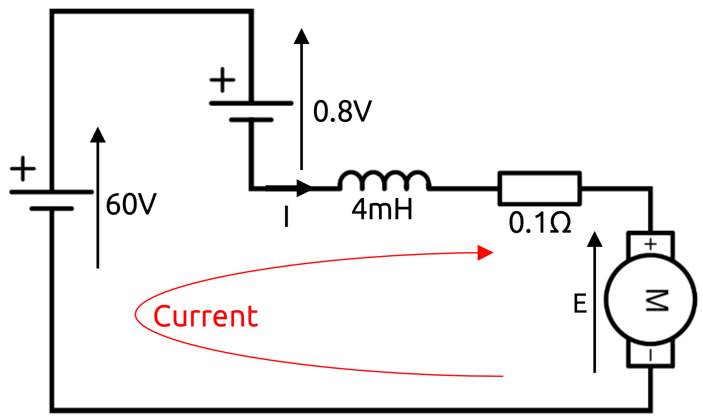


Figure 2: T1 'on' state sub circuit.

Shown below in *figure 3* is the circuit representation for T1 being switched off.

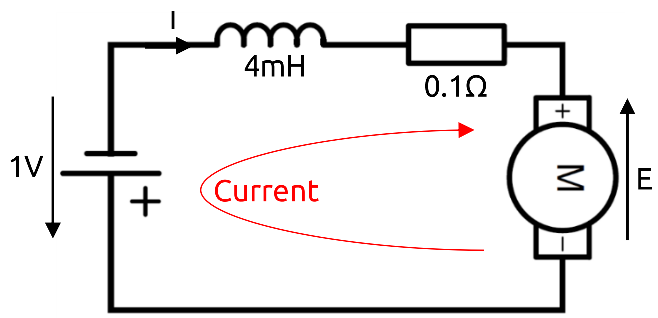


Figure 3: T1 'off' state sub circuit.

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Part B

Firstly, using equation (1), the speed of the motor is converted from rpm to radians per second.

$$\omega = rpm \cdot \frac{2\pi}{60} \tag{1}$$

$$2000 \cdot \frac{2\pi}{60} = \frac{200\pi}{3}\ rads^{-1}$$

The back EMF of the motor, E, is then calculated at this speed using equation (2).

$$E = \psi\omega \tag{2}$$

$$0.08 \times \frac{200\pi}{3} = \frac{16\pi}{3} V$$

$$E \approx 16.76V$$

To calculate the duty cycle, we can use equation (3) to see how the average output voltage varies as the duty cycle is varied, which takes account for the negative voltage across the diode during the off-period.

$$Va = \frac{1}{T}\int_0^T V\,dt$$

$$Va = \frac{1}{T}\left(V_p \cdot t_{on} - V_d \cdot t_{off}\right) \tag{3}$$

(V_p = max voltage, V_d = diode voltage drop)

Rearranging to find t_{on}.

$$Va \cdot T = \left(59.2 \cdot t_{on} - t_{off}\right)$$

$$since\ T = t_{on} + t_{off}$$

$$Va \cdot T = 59.2 \cdot t_{on} - (T - t_{on})$$

$$Va \cdot T + T = t_{on}(59.2 + 1)$$

$$t_{on} = \frac{T(Va + 1)}{59.2 + 1}$$

Since the motor is under no load, the applied voltage, Va, will be equal to the back EMF of the motor, E.

Substituting in values.

$$t_{on} = \frac{0.25 \times 10^{-3}(16.76 + 1)}{60.2}$$

$$\boldsymbol{t_{on} = \tau = 73.73\mu s}$$

The duty cycle can also be calculated.

$$\boldsymbol{duty\ cycle = \frac{\tau}{T} = 29.5\%}$$

Part C

Shown below in *figure 4* is the voltage across the diode as T1 is switched, the maximum and minimum voltages are labelled as well as the switching times.

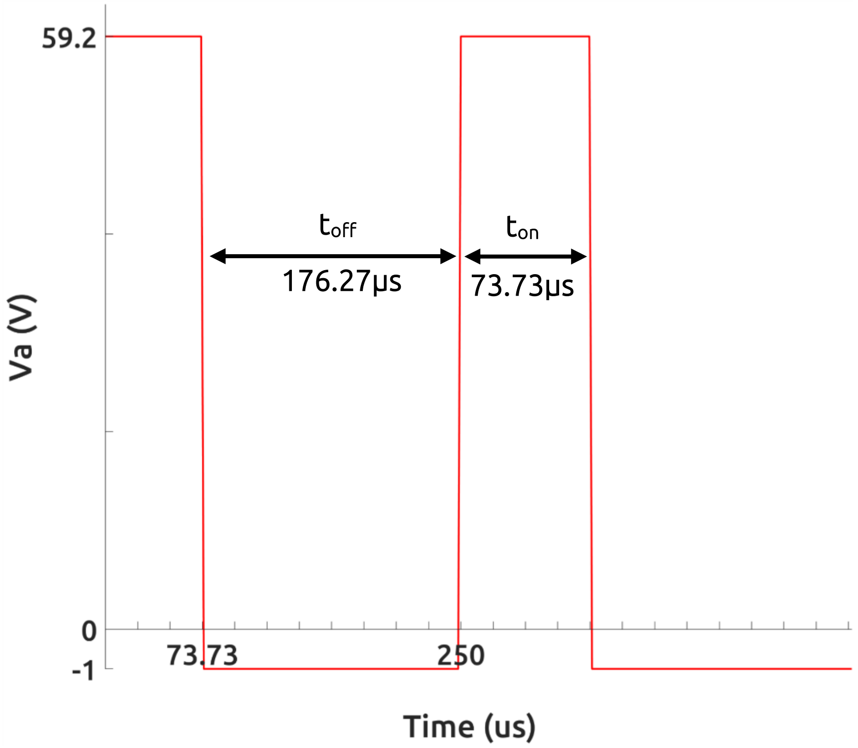

Figure 4: Voltage across D1 as T1 switches.

Part D

The current required to provide the 0.2Nm of torque can be calculated using equation (4).

$$T = \psi I \tag{4}$$

$$\boldsymbol{I = \frac{0.2}{0.08} = 2.5A}$$

Since a load has now been applied, the duty cycle must be varied to provide the necessary voltage to the motor.

The new t_{on} period can be calculated via equation (5).

$$I_{AVERAGE} = \frac{\frac{1}{T}\left(V_p \cdot t_{on} - V_d \cdot t_{off}\right) - E}{R_a} \tag{5}$$

Rearranging to find t_{on}.

$$T(I_{AVERAGE} \cdot R_a + E) = \left(V_p \cdot t_{on} - V_d \cdot t_{off}\right)$$

$$since\ T = t_{on} + t_{off}$$

$$T(I_{AVERAGE} \cdot R_a + E) = (59.2 \cdot t_{on} - (T - t_{on}))$$

$$t_{on} = \tau$$

$$\tau_{new} = \frac{T(I_{AVERAGE} \cdot R_a + E) + T}{60.2}$$

Substituting in values.

$$\tau_{new} = \frac{0.25 \times 10^{-3}(2.5 \cdot 0.1 + 16.76) + 0.25 \times 10^{-3}}{60.2}$$

$$\tau_{new} = 74.77\mu s$$

The peak-peak ripple current can now be calculated using equation (6), where τ is the on-time of T1.

$$\Delta I = \frac{E(T - \tau)}{L_a} \tag{6}$$

$$\Delta I = \frac{16.76(0.25 \times 10^{-3} - 74.77 \times 10^{-6})}{4 \times 10^{-3}}$$

$$\Delta I = 0.734A$$

The minimum and maximum value of the current ripple is calculated by subtracting or adding half of the ripple current to the average current as shown below.

$$\boldsymbol{I_{min} = 2.5 - \frac{0.734}{2} = 2.133A}$$

$$\boldsymbol{I_{max} = 2.5 + \frac{0.734}{2} = 2.867A}$$

Part E

This question did not make the operating conditions of the motor clear in regards to whether or not the motor has a load applied, or if the speed can exceed 800RPM, therefore I have **2 different approaches** with different assumptions.

Approach 1

The first approach assumes that the motor is only running at **800RPM**, with **no load** applied to it.

Using equation (1), the speed is converted from rpm to radians per second.

$$\omega = 800 \cdot \frac{2\pi}{60} = \frac{80\pi}{3} rads^{-1}$$

The back EMF is then calculated for this given speed using equation (2).

$$E = 0.08 \cdot \frac{80\pi}{3} = 6.70V$$

Assuming there is no load on the motor, at a speed of 800rmp, the back EMF is equal to the applied voltage.

$$E = \frac{1}{T}\left(V_p \cdot t_{on} - V_d \cdot t_{off}\right)$$

Rearranging to find the duty cycle.

$$E = \frac{1}{T}\left(V_p \cdot \tau - (T - \tau)\right)$$

$$E = 60.2 \cdot \frac{\tau}{T} - 1$$

$$\frac{\tau}{T} = \frac{6.7 + 1}{60.2}$$

$$\boldsymbol{Duty\ cycle = \frac{\tau}{T} = 12.794\%}$$

Since the duty cycle that causes the motor to spin at 800RPM is known, the time period can be calculated by rearranging equation (7)

$$\Delta I = \frac{V_p - E}{L_a} \cdot \tau \tag{7}$$

$$since\ \mathrm{\tau} = 0.12794 \cdot \mathrm{T}$$

$$T = \frac{\Delta I \cdot L_a}{0.1279(V_p - E)}$$

$$T = 357.32\mu s$$

Therefore, the reciprocal of this is the minimum switching frequency.

$$\boldsymbol{f_{min} = \frac{1}{357.32 \times 10^{-6}} = 2.799kHz}$$

Approach 2

The second approach to this question finds the switching frequency that will ensure the peak-peak ripple current **does not exceed 0.6A** for **any working conditions** above 800RPM.

Firstly, the duty cycle that causes the maximum ripple current to occur is calculated, representing **worst case conditions**.

Temporarily assuming there is no load on the motor, the back EMF of the motor is equal to the applied voltage.

$$E = \frac{1}{T}\left(V_p \cdot t_{on} - V_d \cdot t_{off}\right)$$

This relationship can be substituted into equation (6).

$$\Delta I = \frac{\frac{(T-\tau)}{T}\left(V_p \cdot t_{on} - V_d \cdot t_{off}\right)}{L_a} \tag{8}$$

Simplifying.

$$\Delta I = \frac{\frac{(T-\tau)}{T}(60.2 \cdot \tau - T)}{L_a}$$

$$\Delta I = \frac{-60.2 \cdot \tau^2 + \tau(60.2 \cdot T - T) - T^2}{T \cdot L_a}$$

This formula is differentiated with respect to τ, so that the largest change of current can be calculated.

$$\frac{d\Delta I}{d\tau} = \frac{-100.4\tau + (60.2 \cdot T - T)}{T \cdot L_a}$$

The differential is set to zero to find the duty cycle that causes the maximum change in ripple current.

$$0 = -120.4\tau + 60.2 \cdot T - T$$

$$\frac{\tau}{T} = \frac{61.2}{120.4}$$

$$\boldsymbol{Duty\ cycle = \frac{\tau}{T} \approx 0.5083}$$

Assuming that the motor is operating at the duty cycle of 50.83%, and that there is an **external load bringing the speed down to 800RPM**, equation (9) can be used to find the time period that causes a maximum current ripple of 0.6A.

$$\Delta I = \frac{\tau(V_p - E)}{L_a}$$

$$since\ \tau = 0.5083 \cdot T$$

$$T = \frac{\Delta I \cdot L_a}{0.5083(V_p - E)} \quad (9)$$

Substituting the maximum ripple current of 0.6A, the EMF at 800RPM and the inductance into equation (8).

$$T = \frac{0.6 \cdot 4 \times 10^{-3}}{0.5083(59.2 - 6.7)}$$

$$T = 89.938\mu s$$

The reciprocal of this is calculated to find the minimum switching frequency.

$$\boldsymbol{f_{min} = \frac{1}{89.938 \times 10^{-6}} = 11.119kHz}$$

The graph in figure 5 proves that at a duty cycle of 50.83%, the peak-peak ripple current **never exceeds 0.6A** from the full range of 800RPM - 7000RPM.

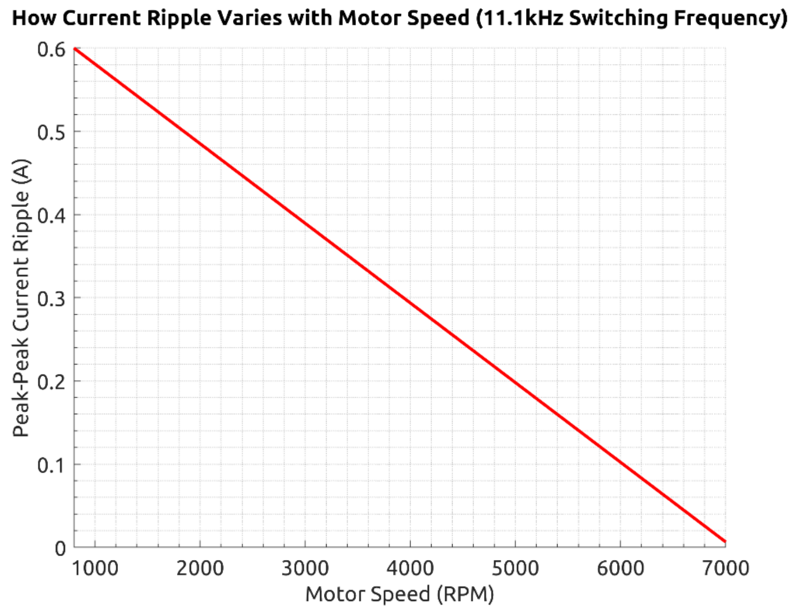

Figure 5: Ripple current vs motor speed.

Part F

The maximum torque of the motor is found when the speed is zero, and the maximum speed is found with the torque is zero. I am not neglecting the semiconductor voltage drops.

Firstly, to find the maximum torque as the duty cycle is varied, the average voltage applied to the motor at each duty cycle is calculated using equation (9). Using Ohms law and the winding resistance, the average current flowing through the motor can then be found, allowing the maximum torque to also be calculated using equation (4). Finally, the maximum speed at each duty cycle can be calculated using equation (2).

The results are shown in *figure 6*.

	Duty Cycle			
	100%	***75%***	***50%***	***25%***
Average Applied Voltage (V)	59.20	44.15	29.10	14.05
Average Current (A)	592.0	441.5	291.0	140.5
Peak Torque (Nm)	47.36	35.32	23.28	11.24
Maximum Speed (RPM)	7066	5270	3474	1677

Figure 6: Calculations for torque-speed characteristics graph.

Due to the current limit of 50A, the maximum torque can be calculated.

$$T = 0.08 \times 50$$

$$\boldsymbol{T_{max} = 4Nm}$$

The torque-speed characteristics graph is shown in *figure 7*, with a cut off at 4Nm to represent the current limit and the shaded boxes representing the rated operating region of the motor.

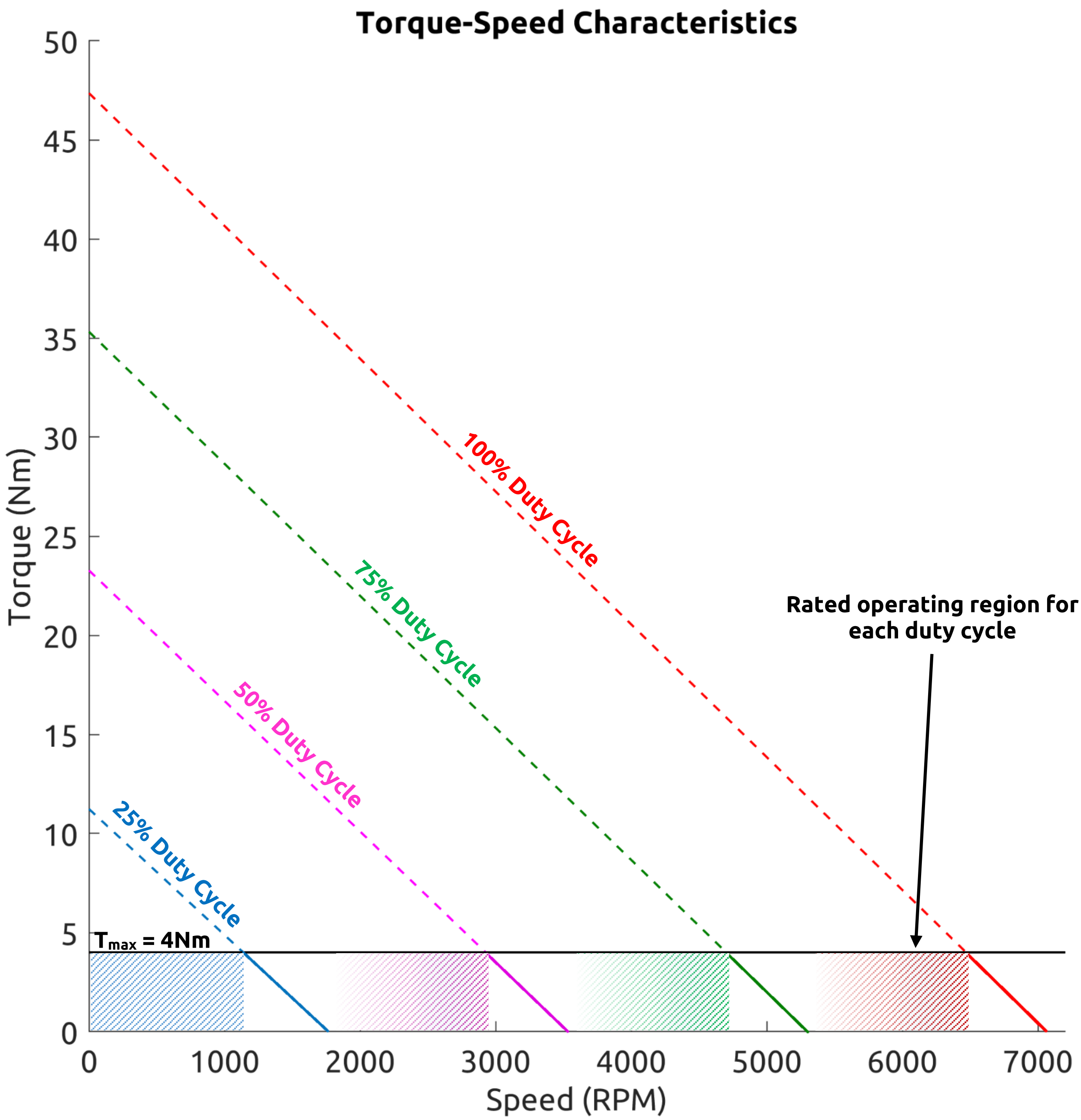

Figure 7: Torque/Speed characteristics graph.

Part G

The circuit shown in *figure 8* is a H-Bridge, which allows the motor to operate in all the 4 quadrants, enabling the motor to consume power and generate power in both directions.

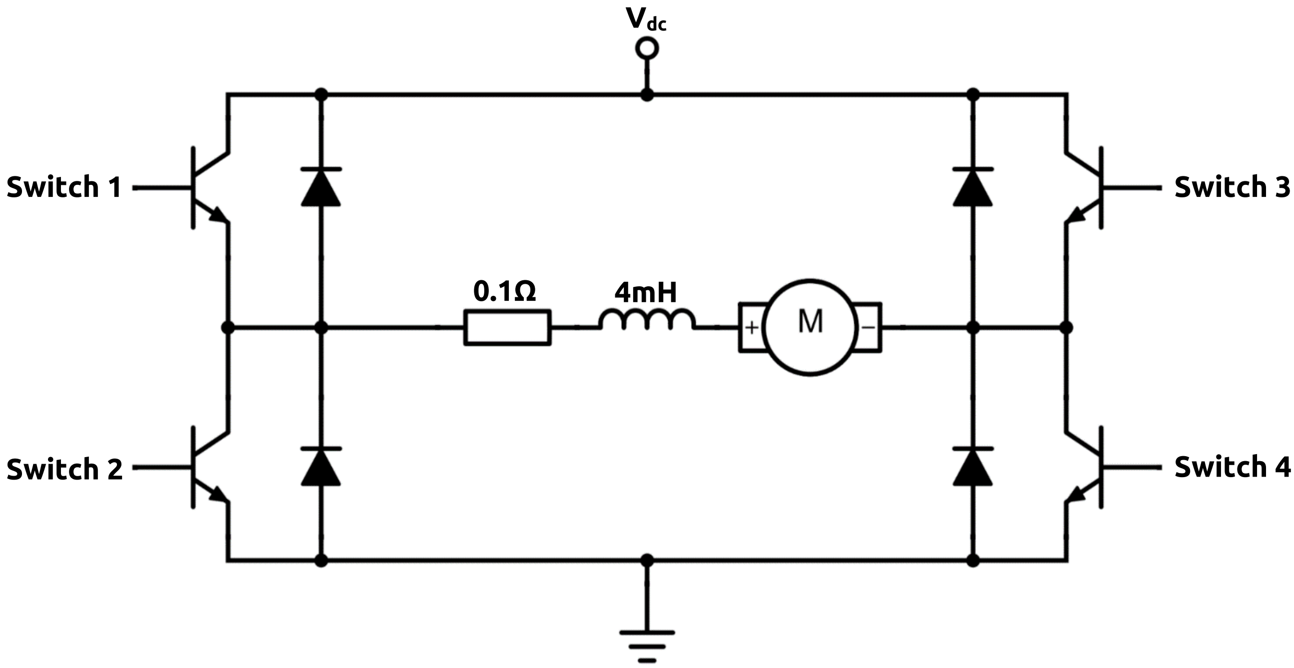

Figure 8: H-Bridge circuit.

Assumptions:

- V_{dc} = 60V
- Duty Cycle for max current ripple = 50% (for average current = 0)
- Switching frequency = 4kHz (T = 0.25ms)
- Transistor voltage drop = 0.8V
- Motor is under no load
- Motor has zero speed
- Motor is being 'held' by repeatedly switching between the motors polarities, average current is zero

The maximum current ripple at a duty cycle of 50% can be calculated using equation (10).

$$\Delta I = \frac{(V_p - E)}{L_a} \cdot \tau \tag{10}$$

Due to the transistor voltage drops.

$$V_p = 60 - 2 \cdot 0.8 = 58.4V$$

Due to zero motor speed.

$$E = 0V$$

Substituting into equation (10).

$$\Delta I = \frac{(58.4 - 0)}{4 \times 10^{-3}} \cdot 0.125 \times 10^{-3}$$

$$\Delta I = 1.825A$$

$$\boldsymbol{I_{min} = -0.9125A}$$

$$\boldsymbol{I_{max} = 0.9125A}$$

A graphical representation of the motor current is shown in *figure 9*.

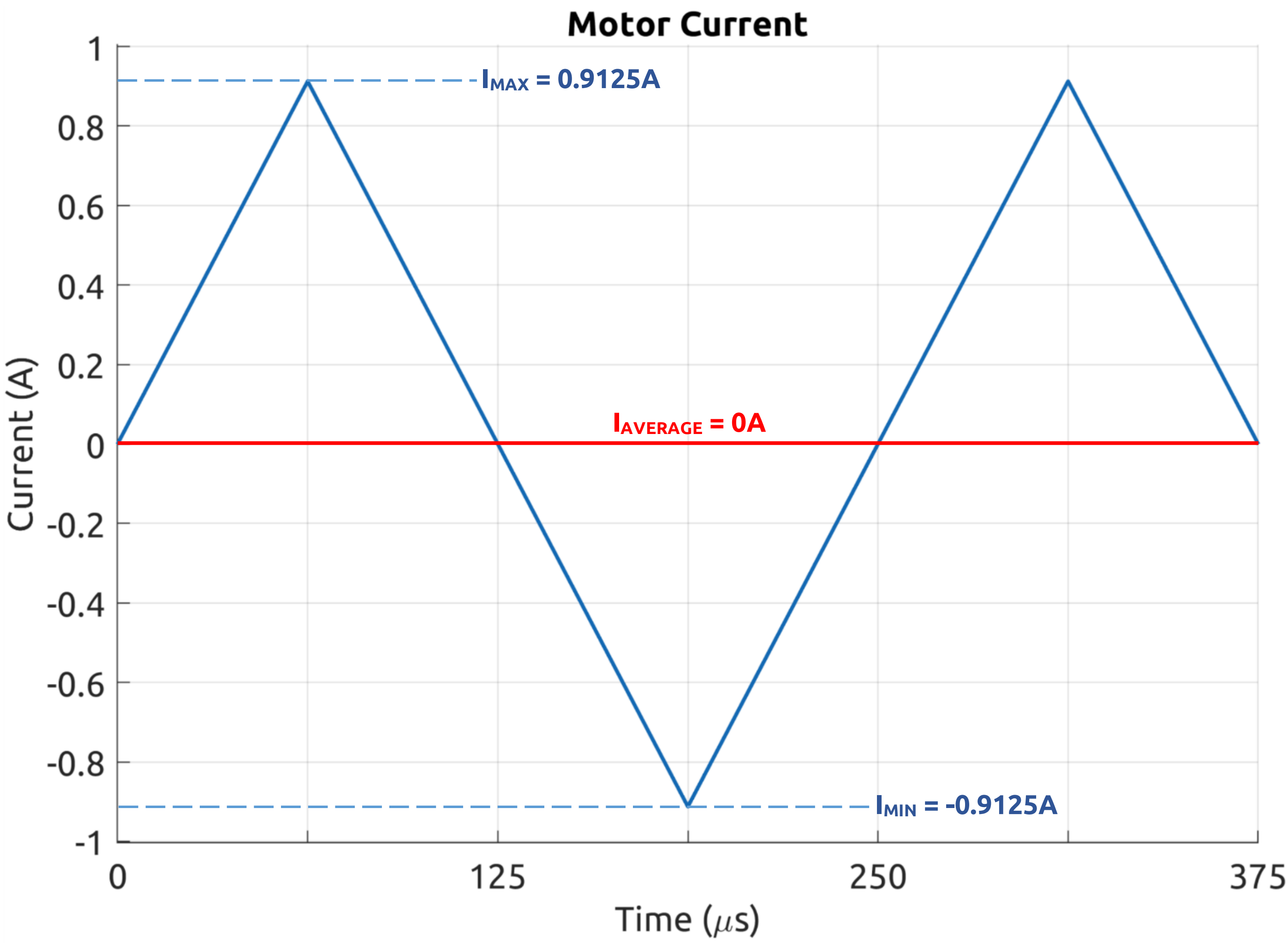

Figure 9: Motor Current.