Benjamin Griffiths (160159871)

EEE223 ASSIGNMENT 3

Part 1

For a coil carrying a current, the relationship between the current flowing through the coil (I), with the flux linkage (ψ) is shown in *figure 1*. Since we are looking for the energy stored in a non-saturated, linear magnetic system, we will only focus on the linear region of this graph, shown in *figure 2*.

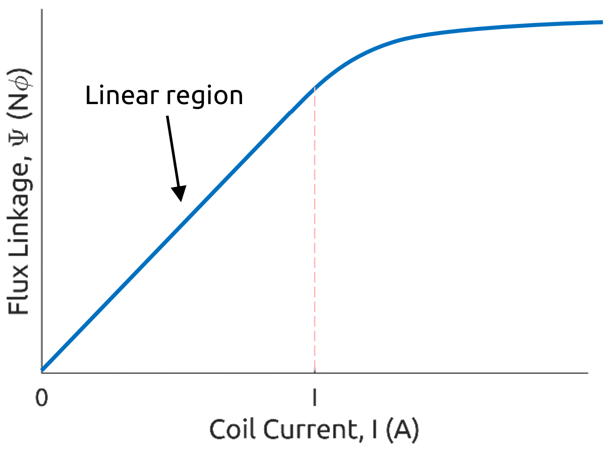

Figure 1: Ψ vs. I graph.

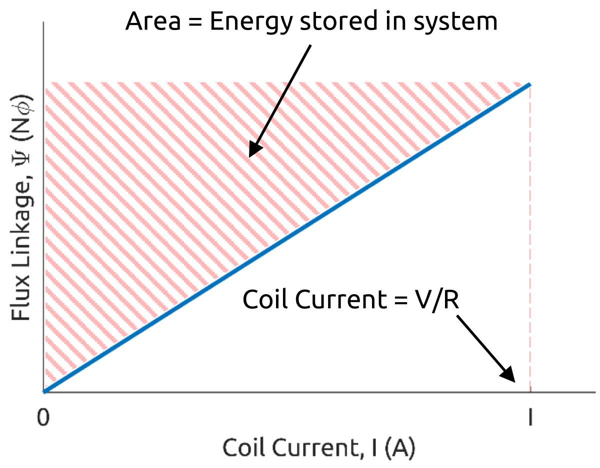

Figure 2: Linear region.

The inductance of the coil is the gradient of the line given by equation (1).

$$L = \frac{\Psi}{I} \quad (1)$$

The energy stored in the coil (W) is equal to the area between the curve and the Ψ axis, given by equation (2).

$$W = \int_0^I I \, d\Psi \quad (2)$$

$$since\ \Psi = LI$$

$$d\Psi = L \cdot dI$$

Substituting into (2) gives:

$$W = \int_0^I L \cdot I \, dI$$

$$W = \left[\frac{1}{2} \cdot L \cdot I^2\right]_0^I$$

$$\boldsymbol{W = \frac{1}{2} L I^2}$$

Electronic & Electrical Engineering
The University of Sheffield

Part 2

A diagram showing the operation of an electromagnetic relay is shown in *figure 3*, with annotations for each element in the diagram.

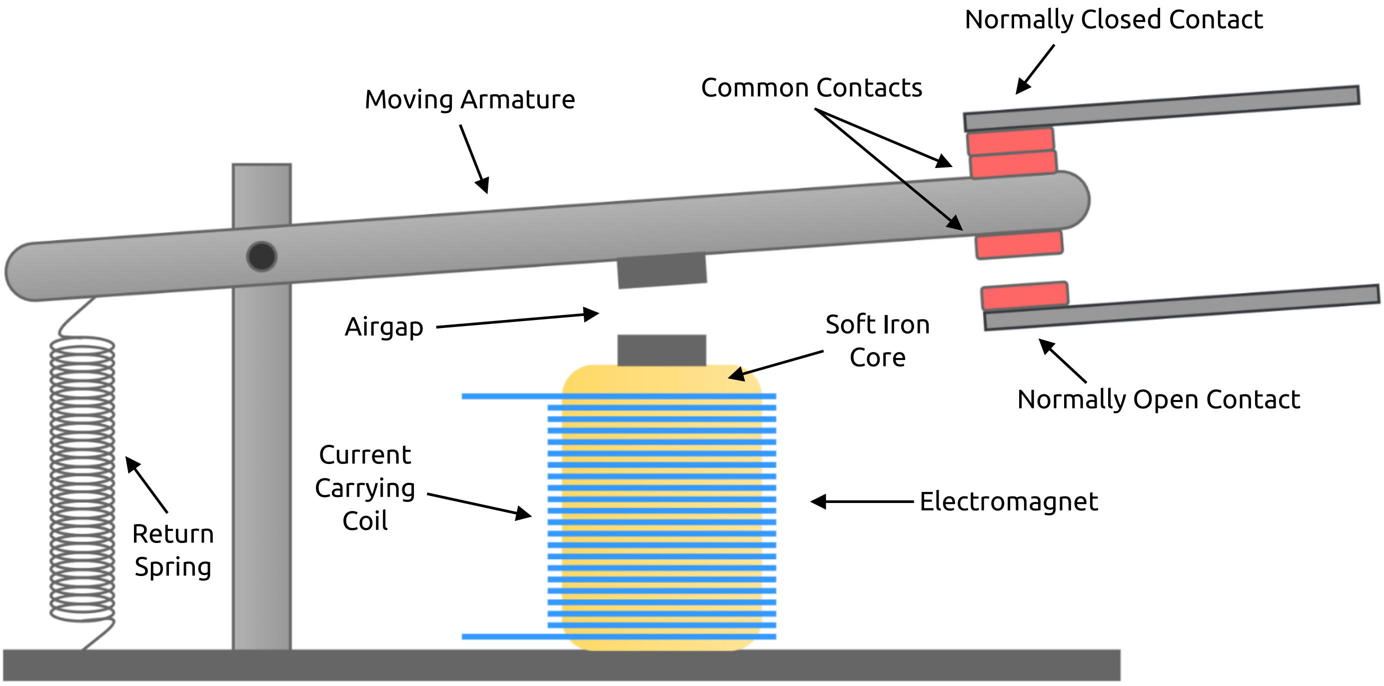

Figure 3: Relay diagram.

When there is no current flowing through the coil in the electromagnet, there is no magnetic attraction between the armature and the electromagnet, hence the force of the return spring brings the common contact on the armature against the normally closed contact.

If a voltage is applied across the electromagnets coil, a current will flow and hence magnetic flux will be produced. This flux will cause a magnetic attraction between the electromagnet and the armature. The force of this attraction will overcome to force of the return spring, hence the armature will move, bringing the common contact and normally open contact together.

The armature will remain in this position as long as there is a current flowing through the coil in the electromagnet. When the current is reduced or stopped, the level of flux produced will drop and the force of the return spring will bring the armature back to its normally closed position.

When the relay is off, the common pin will be electrically connected to the normally closed pin on the relay packaging, and when the relay is activated, the common pin will be electrically connected to the normally open pin.

Part 3

The parameters of the relay in question are shown in *table 1*.

Number of turns (N)	1200 turns
Soft iron cross sectional area (A)	100mm^2
Open position airgap length (x)	5mm
Closed position airgap length (x)	2mm
Spring force (F)	1N

Table 1: Relay specification.

The force of attraction that the electromagnet produces is given by equation (4), with the derivation shown below.

a) $L = \frac{\psi}{I}$ b) $\psi = N\phi$ c) $NI = S\phi$ d) $S = \frac{x}{\mu_0 A}$

Using equation (c) and (d).

$$NI = \frac{x}{\mu_0 A} \cdot \phi$$

Rearranging.

$$\phi = \frac{NI\mu_0 A}{x}$$

Substituting into equation (a).

$$L = \frac{N\phi}{I}$$

$$L = \frac{N^2 \mu_0 A}{x}$$

Using the general formula for force in equation (3).

$$F = \frac{1}{2} I^2 \frac{\mathrm{d}L}{dx} \quad (3)$$

$$\frac{\mathrm{d}L}{dx} = -\frac{N^2 \mu_0 A}{x^2}$$

Substituting into final form.

$$F = -\frac{1}{2} \cdot \frac{I^2 N^2 \mu_0 A}{x^2} \quad (4)$$

$$where\ \mu_0 = \pi \cdot 4 \times 10^{-7}$$

This can be rearranged to equation (5) to find the inductor current I.

$$I = \sqrt{-2 \cdot \frac{x^2 F}{\mu_0 A N^2}} \tag{5}$$

The relays will close when the force of the electromagnet exceed the force of the spring over a gap of 5mm.

$$I = \sqrt{-2 \cdot \frac{(5 \times 10^{-3})^2 \times -1}{\pi \cdot 4 \times 10^{-7} \times 100 \times 10^{-6} \times 1200^2}}$$

$$\boldsymbol{I_{CLOSE} = 0.526A}$$

The relays will open again when the force of the electromagnet falls below the force of the spring over a gap of 2mm.

$$I = \sqrt{-2 \cdot \frac{(2 \times 10^{-3})^2 \times -1}{\pi \cdot 4 \times 10^{-7} \times 100 \times 10^{-6} \times 1200^2}}$$

$$\boldsymbol{I_{OPEN} = 0.210A}$$

Part 4

The current required to open and close the relay is different because the electromagnets force is operating over a different airgap distance. For example, when the relay needs to be closed, the electromagnet has to provide the necessary force over a distance of 5mm, but it only has to provide this force over a distance of 2mm once the relay is closed. Therefore, since the magnetic force produced by the current is proportional to the airgap distance, the current required to initially overcome the springs force and close the contacts will be greater than the current required for the spring to re-open the contacts.

This characteristic of the relay benefits its operation because the armature will only tend to either the closed position or open position, not any position in-between. This is because once the current flowing in the coil is large enough to overcome the spring force, the armature will shift to its closed position, decreasing the airgap and therefore lowing the current requirements. The spring force will only overcome the electromagnets force once the current is reduced significantly, making the operation of the relay reliable and predictable, because the armature will always 'snap' to either of the positions.

If the current to open and close the contacts was the same, the two states would be fighting over each other during the transition period. For instance, if the armature was in-between the two states, the current at which the electromagnets force can overcome the springs force is equal to the current at which the spring force can overcomes the electromagnets force. Hence, small fluctuations in current would determine which position the armature tends to, making the operation unstable and uncertain.

Part 5

The relay driver circuit shown in *figure 4* was simulated in LTSpice using as close to ideal as possible components for the diode and transistor as seen in *figure 5*. The transistor was switched using a square wave or suitable length to allow for the currents to settle after each switching event.

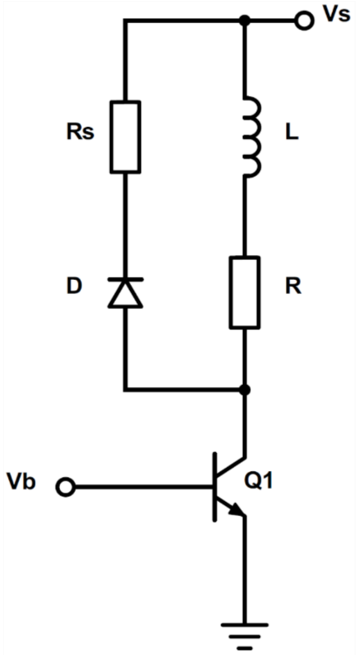

Figure 4: Relay driver circuit Schematic.

Element	Value
V_S	48V (DC)
R	30Ω
L	0.06H
I_{ON}	0.5A
I_{OFF}	0.2A
V_D	0V
$V_{CE(ON)}$	0V

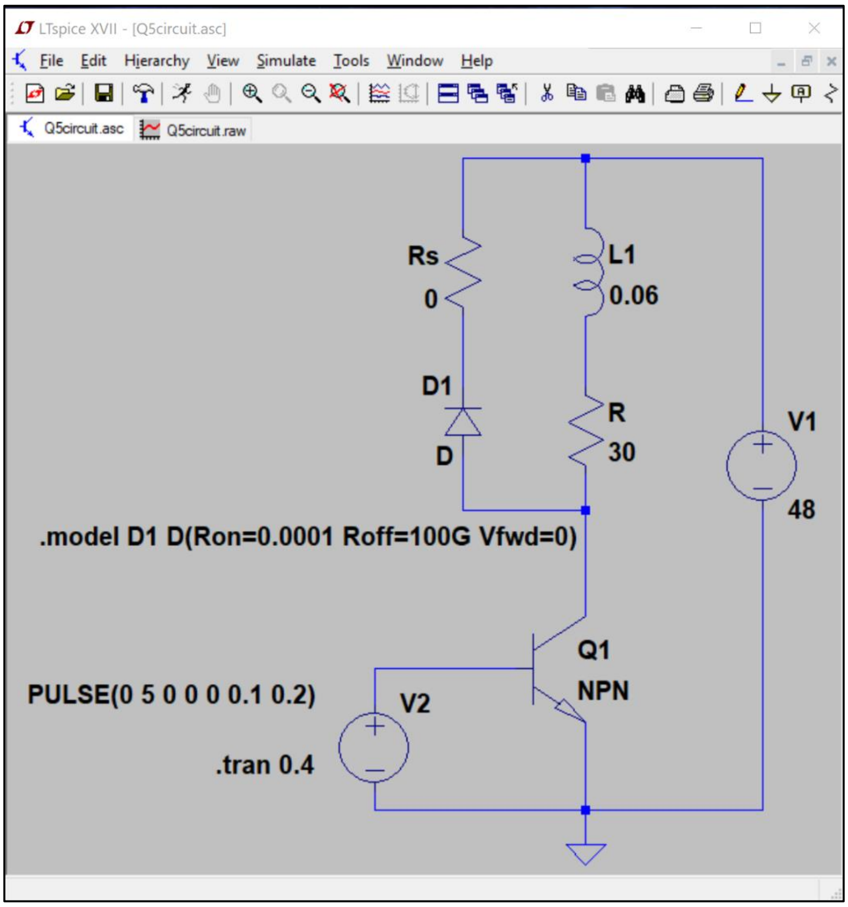

Figure 5: LTSpice model.

The waveforms for the current flowing through the inductor (I_L), resistor (I_R) and diode (I_D) as the transistor is turned on and off are shown in *figure 6.*

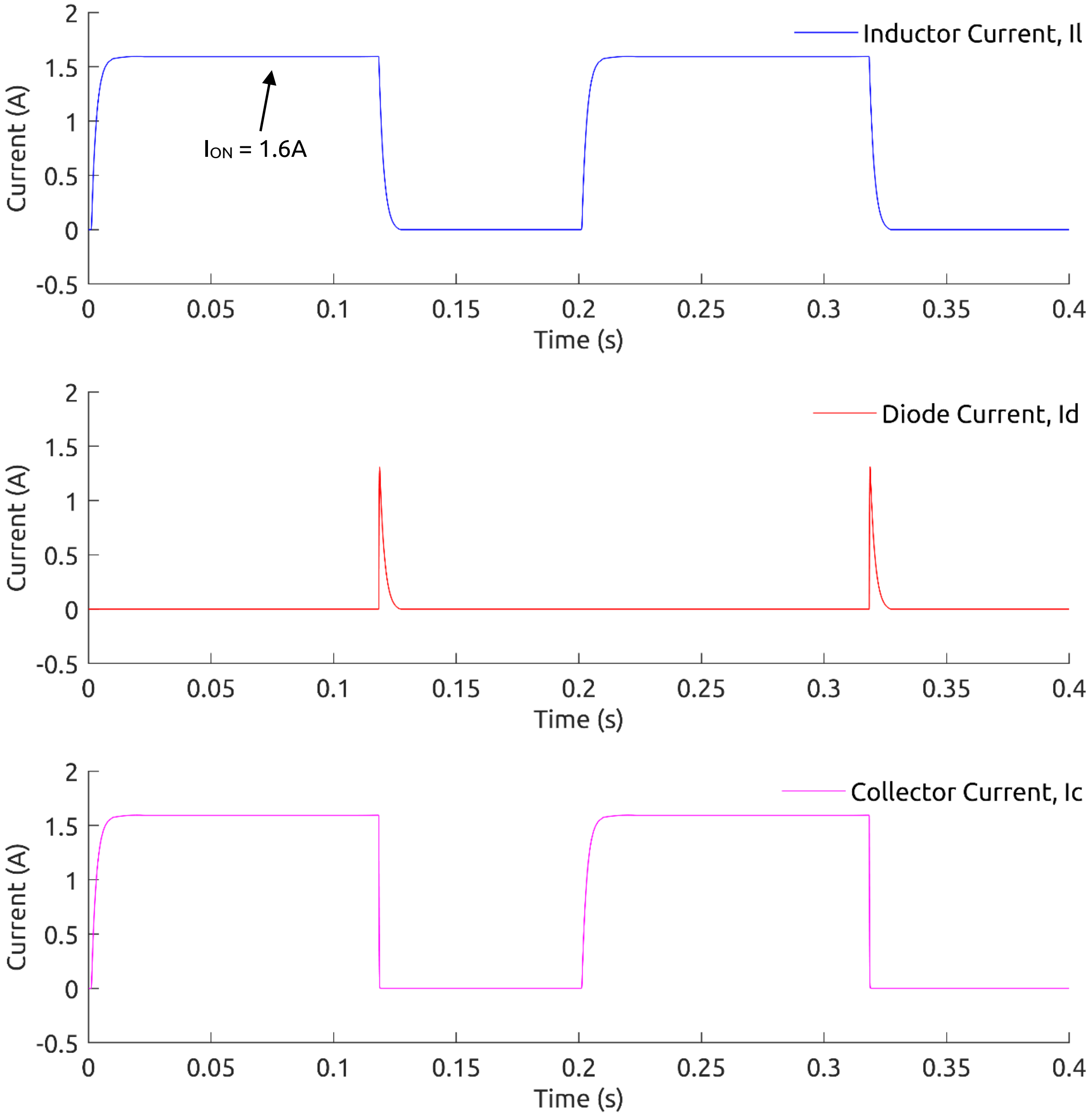

Figure 6: Current Waveforms.

Part 6

To find the time is takes for the relay to respond, the time it takes for the inductor current to rise to 0.5A must be calculated, seen in *figure 7.*

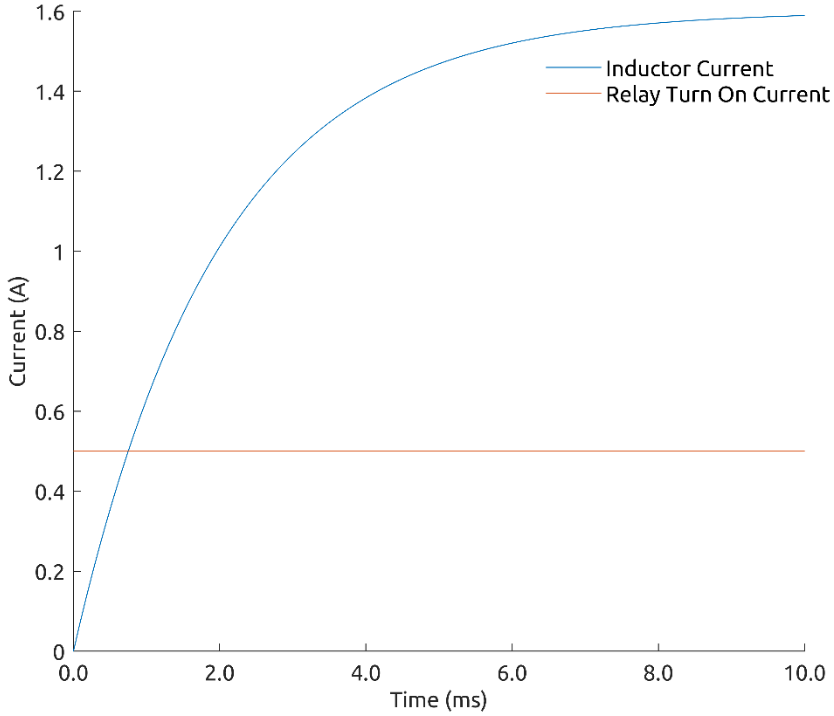

Figure 7: Turn on current waveform.

The time at which the 2 lines intercept and the relay turns on is defined in equation (6).

$$I_L(t) = \frac{V_S}{R} \cdot \left(1 - e^{\frac{-t}{\tau}}\right) \quad (6)$$

$$where\ \frac{V_S}{R} = steady\ state\ current$$

$$where\ \tau = \frac{L}{R} = time\ constant$$

Rearranging to find t.

$$t = -\tau \cdot \ln\left(1 - \frac{I_{ON}}{V_S/_R}\right)$$

$$\tau = \frac{0.06}{30} = 2ms$$

$$\frac{V_S}{R} = \frac{48}{30} = 1.6A$$

$$t = -0.002 \cdot \ln\left(1 - \frac{0.5}{1.6}\right)$$

$$\boldsymbol{t = 0.749ms}$$

Part 7

To find the time is takes for the relay to turn off, the time it takes for the inductor current to fall to 0.2A must be calculated, seen in *figure 8.*

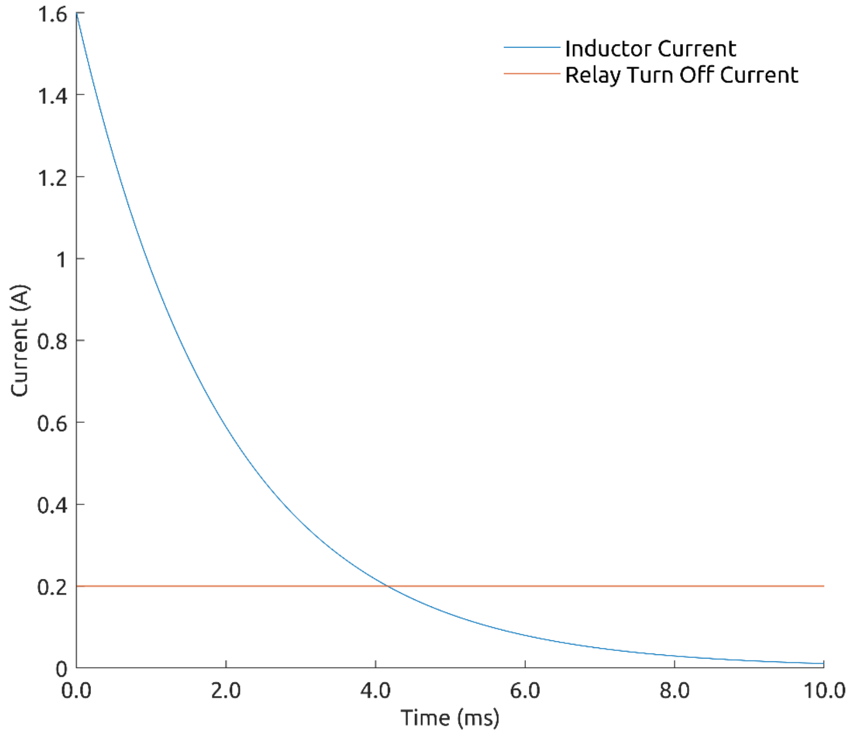

Figure 8: Turn off current waveform.

The time at which the 2 lines intercept and the relay turns off is defined in equation (7).

$$I_L(t) = \frac{V_S}{R} \cdot e^{\frac{-t}{\tau}} \tag{7}$$

$$where\ \frac{V_S}{R} = steady\ state\ current$$

$$where\ \tau = \frac{L}{R} = time\ constant$$

Rearranging to find t.

$$t = -\tau \cdot \ln\left(\frac{I_{OFF}}{V_S/_R}\right)$$

$$\tau = \frac{0.06}{30} = 2ms$$

$$\frac{V_S}{R} = \frac{48}{30} = 1.6A$$

$$t = -0.002 \cdot \ln\left(\frac{0.2}{1.6}\right)$$

$$\boldsymbol{t = 4.159ms}$$

Part 8

Since the time constant is dependent on the total resistance in the series circuit, the time constant for a finite value of R_s can be defined by equation (8).

$$time\ constant = \tau = \frac{L}{R + R_S} \tag{8}$$

Equation (?) can be manipulated to find the time constant required to give a turn off time of 2ms.

$$\tau = \frac{-t}{\ln\left(\frac{I_{OFF}}{V_S/R}\right)}$$

Substituting in t=2ms.

$$\tau = \frac{-0.002}{\ln\left(\frac{0.2}{1.6}\right)}$$

$$\tau = 0.962ms$$

This value can now be substituted back into equation (8) to find the value R_S.

$$R_S = \frac{L}{\tau} - R$$

$$R_S = \frac{0.06}{0.962 \times 10^{-3}} - 30$$

$$\boldsymbol{R_S = 32.383\Omega}$$

Part 9

To find the minimum V_{CE} rating of the transistor, the largest voltage that will be applied across it must be calculated. The largest voltage across the transistor will occur at the instant the transistor is switched off. The current flowing through the inductor (1.6A) will be routed through the diode (D) and resistance (Rs) network, hence the voltage at the collector will be positive with respect to the supply voltage.

$$Max\ Collector\ Voltage = V_S + I_{LR} \cdot R_S$$

$$\boldsymbol{Max\ Collector\ Voltage = 48 + 1.6(39) = 110.4V}$$

Therefore, the transistor should ideally have a maximum V_{CE} rating exceeding 120V to avoid damage.

The expected voltage waveform that would be expected at the collector and the base of the transistor is shown in *figure 9.*

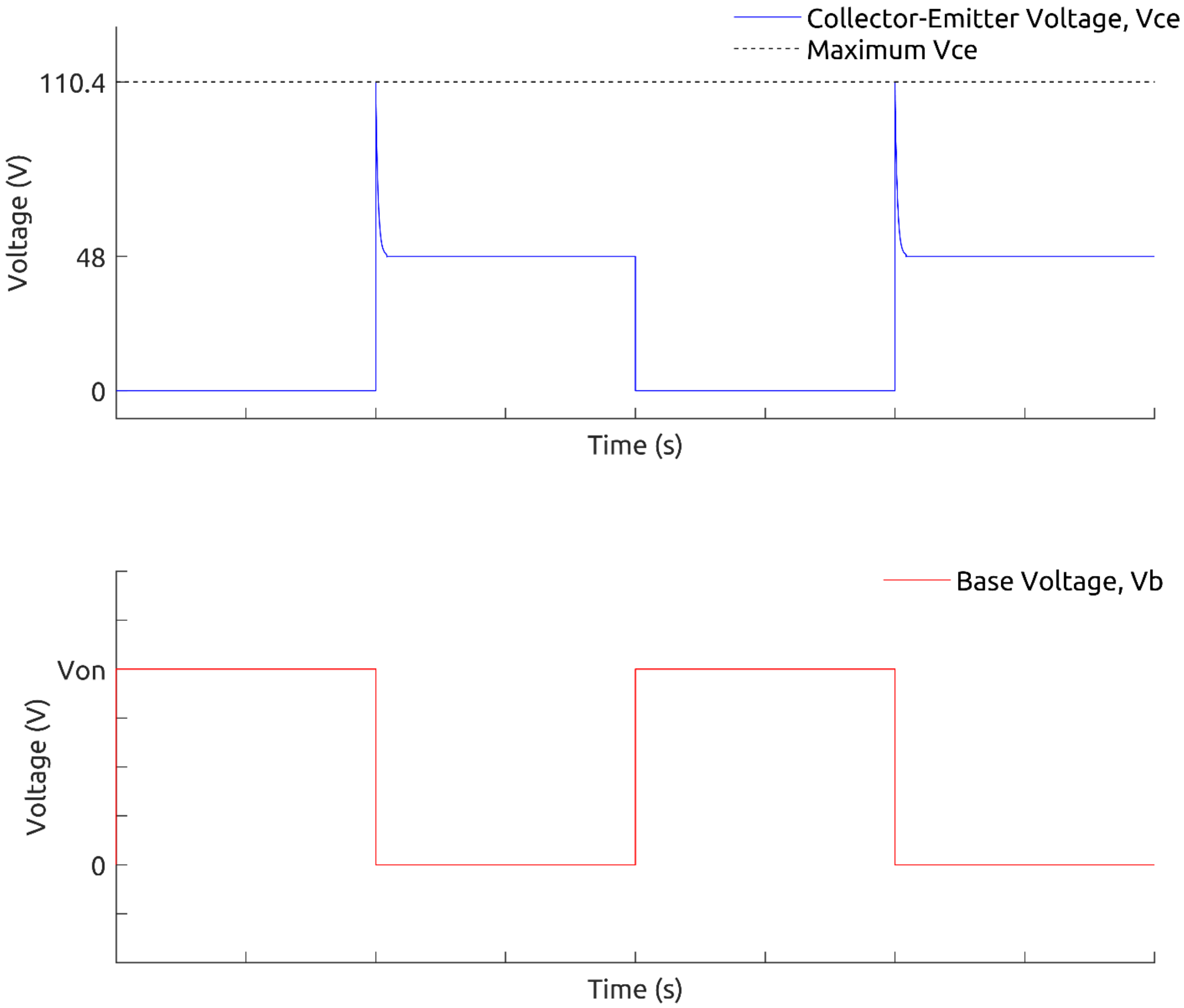

Figure 9: V_{CE} and V_B expected waveforms.